



ISSN 1791-3144

DEPARTMENT OF ECONOMICS
DISCUSSION PAPER SERIES

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Discussion Paper No. 11/2026

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Non-monotonic entry dynamics in oligopoly with common ownership

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July 15, 2026

Abstract

We show how common ownership fundamentally transforms entry in an uncertain market from sequential preemption into a regime of coordination with contest-like dynamics. We do this by studying an industry with three firms facing a stochastically evolving demand that internalize the effect of their entry and output decisions on rivals. Equilibrium exhibits an accordion effect with respect to internalization: common ownership softens the last entry, intensifies competition at the duopoly stage, and ultimately slows down first market entry, reducing overall dynamism. But unlike in two-firm industries, a coordination equilibrium emerges at the duopoly stage if option value is high enough. The contest-like dynamics in this equilibrium accelerate first entry and raise welfare.

JEL Classification: D25, G32, L13.

Keywords: accordion effect; business dynamism; common ownership; investment under uncertainty; preemption games; real options.

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1 Introduction

Macroeconomists have long noted decreasing business dynamism in the United States and other advanced economies, evidenced by declining firm entry rates (Decker, Haltiwanger, Jarmin, & Miranda, 2014). Real options logic could explain this decline had the value of waiting increased over time, but volatility does not appear to have broadly risen aside from transitory peaks during the global financial crisis and the coronavirus pandemic. Decreasing dynamism has instead been related primarily to increasing market concentration and margins (Akcigit & Ates, 2021), along with rising regulatory costs (Gutiérrez & Philippon, 2019), and eventually with the concurrent rise in common ownership (Backus, Conlon, & Sinkinson, 2021). According to this last channel, as competing firms have become more tightly linked through shared ownership ties, entry incentives in many industries may well have decreased alongside product market competition (Vives & Vivasinos, 2025).

We propose in this paper to explicitly relate common ownership and business dynamism by modeling the entry dynamics of a small number of firms subject to common ownership into a market whose demand is driven by a continuously evolving stochastic shock. Firms are initially symmetric and decide when to pay an irreversible entry cost, giving rise to preemptive interactions. Common ownership however makes firms internalize rival payoffs, which softens product market competition but may either increase or decrease entry incentives depending on where firms stand in the entry order. We find the interaction of internalization and preemptive dynamics leads to a rich set of equilibrium outcomes, summarized below in Figure 3. Two distinctive effects emerge from our analysis: common ownership generally operates as a friction, whereby increased rents slow (most) entries, but under high internalization and high uncertainty, a highly different competitive effect emerges whereby firms race aggressively for the first entry slot.

The linkage between internalization and softened product market competition is conceptually straightforward (Reynolds & Snapp, 1986), but the effect of common ownership on entry decisions is more involved, as higher profits from entry into a less competitive market are offset by the perceived erosion of the value of incumbents. Generally, common ownership drives follower firms to set a higher threshold for market entry, which in turn intensifies positional competition. To generate a rich enough pattern of equilibrium outcomes where multiple followers interact while remaining analytically tractable, we focus our analysis on a three-firm industry. The three-firm structure naturally reconciles increased positional competition with lower market dynamism, because a tighter race between the last two firms relaxes the initial race to enter first, slowing most of the industry’s entries. But in addition to this first mechanism common ownership can also give rise to coordinated investment behavior, because the internalized profits from the first firm allow followers to wait more patiently and eventually capture the option value of delay jointly.

Under a mild restriction on how firm numbers and internalization affect product market competition and investment incentives, we derive closed-form value functions for the firms in our model by backward induction. Depending on the intensity of competition between the last two firms, on the option value of investing jointly, and on the last firm’s entry incentive, entries in our model occur either sequentially or in waves, and these waves may consist either of a direct jump to duopoly (if follower competition is sufficiently strong) or a jump from monopoly

to three firms (if option value is high enough), and the industry may also ultimately operate with fewer than three firms (even though the third firm’s entry is privately profitable).

The benchmark if firms are symmetric and in the absence of either internalization or uncertainty is a *Full Sequential* equilibrium where all three firms enter successively. We show that internalization in this case exerts an accordion effect on entries,¹ whereby the first entry is delayed, the second occurs earlier, and the third finally is delayed as well as internalization increases. Our numerical analysis then shows that increasing internalization levels lower expected entries overall, consistently with observed patterns of declining business dynamism.

A novel finding stemming from our analysis is that with high uncertainty, common ownership produces a *Follower Coordination* equilibrium where the last two firms hold off entering, which they ordinarily would not manage to do in a new market model. Such an equilibrium arises if laggard firms are content to both wait, which can occur because perceived payoffs from the leader’s monopoly operation represent a form of dividend, provided that the option value from jointly investing is high. Such coordination by laggard firms paradoxically raises the profitability of entering first, and in the limiting case where the second and third firms abstain from ever investing, competition to be the first firm in the market reduces to a *Monopoly Contest* equilibrium, with a single entry occurring at the breakeven threshold. Such interactions are consistent with empirical evidence that common ownership induces product market coordination (He & Huang, 2017), and also help explain how greater dynamism might correlate with high markups at the industry level (Albrecht & Decker, 2026). Coordination at the duopoly stage leads to earlier first entry into the market than in single-firm or two-firm industries, and we show numerically that such equilibria are likelier to arise as demand curvature decreases. Equilibria with coordination at the duopoly stage have important normative implications, as they produce higher consumer surplus and welfare than in the absence of common ownership, contributing a novel perspective to the discussion concerning whether and how common ownership should be regulated (Posner, Scott Morton, & Weyl, 2016).

Even without such coordination, heightened competition at the duopoly stage still puts pressure on the race to enter first. We find that with sufficient internalization and uncertainty, duopoly competition is high enough that a *Clustering* equilibrium arises where the two first firms enter simultaneously. Without common ownership, such an equilibrium would not arise with symmetric firms (Matros, Smirnov, & Wait, 2024). Competition between laggards in our model therefore generates two different kinds of “investment waves”, either optimal simultaneous investments if the second and third firms succeed in coordinating or clustering between the first two if laggard competition is intense enough.

In addition to these outcomes finally, it can occur in our model that the last firm’s investment incentive takes negative values even though its entry is privately profitable, resulting in either a *Sequential Duopoly* equilibrium instead of Full Sequential, or a *Duopoly Contest* equilibrium instead of Clustering.

¹(Bouis, Huisman, & Kort, 2009) describe the accordion effect as: “an exogenous demand shock results in a change of the wedge between investment thresholds of the first and second investor that is qualitatively different from the change of the wedge between the second and third investment threshold.”

1.1 Related literature

Beyond the contributions discussed in the introduction, this paper relates broadly to work on common ownership, which has relevance for both US and EU firms (Boot, Seldeslachts, & Banal-Estañol, 2022), and whose potential for anticompetitive behavior in specific industries was brought to light by (Azar, Schmalz, & Tecu, 2018). Further research has highlighted both negative and positive effects of internalization on R&D investments, depending on the level of knowledge spillovers (López & Vives, 2019; Antón, Ederer, Giné, & Schmalz, 2025). Our work complements this stream by highlighting how negative and positive effects arise even without spillovers. In particular, softened product market competition due to common ownership can exacerbate dynamic competition, resulting in earlier entry though if internalization softens product market competition sufficiently, the effect can be reversed (Zormpas & Ruble, 2021). Relative to this work and to (Vives & Vivasinos, 2025) which studies entry among firms with symmetric entry incentives, our work here captures richer dynamic patterns that are possible only if more than two firms are brought into the picture.

Models of entry timing with three or more firms that have highlighted novel strategic effects relative to the duopoly case include (Bouis et al., 2009), which shows that the first two firms may enter simultaneously or cluster their investment if subsequent duopoly competition between the remaining firms is sufficiently strong, and that parameter changes may affect successive firms in opposing ways. More recent contributions to this literature include (Faninam, Huisman, & Kort, 2023), which incorporates capacity choice alongside timing decisions, and (Matros et al., 2024), which relates clustering to sunk cost asymmetry in a deterministic model.

The remainder of the paper is organized as follows. Section 2 lays out the assumptions of the model. Section 3 studies the equilibrium of the model. Section 4 provides additional numerical insights into the properties of the model. Section 5 concludes.

2 The model

We consider an oligopolistic industry consisting of three identical firms that have the ability to produce a homogeneous good. The firms are initially inactive but can enter the market at any time by paying a sunk cost $I > 0$. Undertaking the investment gives access to a profit flow $Y(t) D_k$ where D_k is the baseline cash flow, $k \in \{1, 2, 3\}$ is the number of active firms and $Y(t)$ is a stochastic process that measures the current market size. $Y(t)$ follows a geometric Brownian motion

$$dY(t) = \mu Y(t) dt + \sigma Y(t) dw(t) \quad (1)$$

with starting value $Y(0) = y$ where $dw(t)$ is the increment of a standard Wiener process. We suppose that Y is low enough that firms initially prefer to wait rather than invest. The firms are risk-neutral, value-maximizing, and discount cash flows at the same constant rate r . As is standard in the investment literature we assume $r > \mu$ in order to ensure payoff convergence.

The structure of this industry involves symmetric common ownership. As a result, each firm internalizes a share $\lambda \in [0, 1]$ of the cash flows of its competitors. The following stylized example provides a simple microfoundation for such a pattern of internalization. Label the three firms A , B and C and denote the net present values of their cash flows by V_j^0 , $j \in \{A, B, C\}$. Suppose that the three firms are each held by n owners where $s_i^j \in [0, 1]$ denotes owner i 's share of firm

j with $\sum_{i=1}^n s_i^j = 1$, $j \in \{A, B, C\}$. Of the n owners of each firm, m are fully diversified owners who hold stakes in all three firms and $n - m$ are specialized owners with stakes in one firm only. The wealth of a specialized owner i of firm j is $s_i^j V_j^0$, and the wealth of a diversified owner i is $s_i^A V_A^0 + s_i^B V_B^0 + s_i^C V_C^0$.

In the absence of agency frictions, firm j maximizes the aggregate wealth of its owners. Its objective function is therefore

$$\sum_{i \in T} (s_i^A V_A^0 + s_i^B V_B^0 + s_i^C V_C^0) + \sum_{i \in S} s_i^j V_j^0 = V_j^0 + \sum_{i \in T} s_i^k V_k^0 + \sum_{i \in T} s_i^l V_l^0 \quad (2)$$

where $j, k, l \in \{A, B, C\}$, $j \neq k \neq l$, and T, S are the sets of diversified and specialized owners respectively. For ownership structures with similar aggregate common ownership levels we obtain $\lambda = \sum_{i \in T} s_i^A = \sum_{i \in T} s_i^B = \sum_{i \in T} s_i^C$. The parameter λ is the relative weight each firm places on rival value, or alternatively put, the degree to which it internalizes the effect of its actions on its competitors due to the common ownership structure. In the special case where ownership is equally divided ($s_i^j = 1/n$ for all i), $\lambda = m/n$. If all owners are specialized ($m = 0$) then $\lambda = 0$, which nests the standard profit-maximization case, and if all owners are common ($m = n$) then $\lambda = 1$ and firms maximize total industry value. Empirical estimates of internalization have reported average λ values around 0.7 for U.S. firms and 0.21 for European firms (Backus et al., 2021; Boot et al., 2022).

Symmetric common ownership implies that the firms optimize over perceived objective functions which put a positive weight on rival values. Letting $\pi_j^0(\mathbf{x})$ denote firm j 's profit from a set of actions $\mathbf{x} = \{x_A, x_B, x_C\}$, under common ownership the decision problem for a given firm i is

$$\max_{x_i} [\pi_i^0(\mathbf{x}) + \lambda(\pi_k^0(\mathbf{x}) + \pi_l^0(\mathbf{x}))], \quad i, k, l \in \{A, B, C\}, \quad i \neq k \neq l. \quad (3)$$

2.1 Product market equilibrium cash flows with common ownership

Common ownership affects the level of product market competition in the short run. Product market cash flows therefore depend on both the number of firms and the ownership structure. Let D_1 denote the monopoly cash flow, and $D_N(\lambda)$ the cash flow for $N \geq 2$ firms. $D_N(\lambda)$ is computed as the firm's cash flow in an oligopoly equilibrium in which each of the N firms maximizes the objective in (2), and we suppose it to be a differentiable function of the degree of internalization.

Throughout our analysis we make several other assumptions regarding product market cash flows, starting with the following.

Assumption A (Cash flows). *Instantaneous cash flows satisfy:*

(i) $D_1 \geq 2D_2(\lambda) \geq 3D_3(\lambda) > 0$ and

(ii) $D'_k(\lambda) \geq 0$,

with strict inequalities if $\lambda < 1$.

Assumption A(i) requires that increased competition reduce industry profits, i.e. additional firms have a business-stealing effect. This restriction is satisfied under oligopoly competition between firms producing homogeneous goods with constant returns to scale. The positive cash

flow requirement rules out the extreme forms of price competition. Assumption A(ii) reflects the idea that common ownership gradually softens product market competition, resulting in higher individual profits.

In deciding entry, firms respond to an instantaneous investment incentive consisting of their perceived cash flows upon entry net of the opportunity cost arising from the internalized profits of rival firms. Suppose that there are currently i firms in the market and that entry raises the number of firms to j . The change in an entrant's perceived profit is

$$\Delta_{j,i}(\lambda) := D_j(\lambda) - \lambda(iD_i(\lambda) - (j-1)D_j(\lambda)). \quad (4)$$

The first term is the firm's own baseline profit flow increase from entry and the second term is the perceived effect on rival firms, which is negative. Several investment incentives recur in our analysis. We denote them by $\Delta_2(\lambda) := \Delta_{2,1}(\lambda)$ (second firm incentive), $\Delta_3(\lambda) := \Delta_{3,2}(\lambda)$ (third firm incentive), and $\Delta_2^S(\lambda) := \Delta_{3,1}(\lambda)$ (joint incentive of second and third firms).

A first restriction, common in entry timing games and in the technology adoption literature, is that the investment incentive decrease with the firm's rank in the entry order. Assumption A(i) directly implies $\Delta_2(\lambda) = (1+\lambda)D_2(\lambda) - \lambda D_1 < D_1 = \Delta_1$, and we require the same property to hold for the second and third firms.

Assumption B (Investment incentive). *The investment incentives of the second and third firms satisfy $\Delta_2(\lambda) > \Delta_3(\lambda)$.*

Investment incentives need not be everywhere positive. Under Cournot competition with homogeneous goods, Assumption A implies $\Delta_2(\lambda)$ and $\Delta_3(\lambda)$ take positive values around zero and just below one, so negative values may only arise inside the unit interval.²

Increases in internalization affect investment incentives (4) through a variety of channels. First of all, the firm's own cash flow $D_j(\lambda)$ increases, but in addition to this there is an impact on the internalized profit of rival firms $\lambda(iD_i(\lambda) - (j-1)D_j(\lambda))$ which is generally ambiguous. For our sensitivity analysis we make the following additional restriction regarding the sensitivity of investment incentives to common ownership.

Assumption C (Internalization and investment incentives). *The investment incentives of the second and third firms satisfy $\Delta_2'(\lambda), \Delta_3'(\lambda) < 0$.*

Assumption C requires that an increase in the degree of internalization λ weakens the instantaneous investment incentives $\Delta_2(\lambda)$ and $\Delta_3(\lambda)$. Given that in the limiting case $\lambda = 1$ the objectives of all firms coincide so $2D_2(1) = 3D_3(1) = D_1$ (Assumption A(i)), we have $\Delta_2(1) = \Delta_3(1) = 0$ so Assumption C implies $\Delta_2(\lambda), \Delta_3(\lambda) > 0$ for $\lambda < 1$. Unless all owners are common therefore, potential entrants have an incentive to enter the market and this incentive weakens as λ increases.

Assumption C is a stronger assumption that we do not require for the main equilibrium analysis. It is nevertheless satisfied for λ close to zero, so a first motivation for such a restriction is that it is consistent with the baseline situation where firms do not internalize at all. In addition, this condition is satisfied in standard oligopoly models like Cournot competition with homogeneous goods and constant unit cost under a broad range of demand specifications.

²We have $\Delta_i(0) = D_i(0) > 0$ and $\Delta_i(1) = 0$, and as we show in Appendix A.1, $D_i'(1) = 0$ so $\Delta_i'(1) = (i-1)(D_i(1) - D_{i-1}(1)) < 0$.

Proposition 1. *If products are homogeneous, unit costs are constant, and firms compete in quantities, then Assumption C holds so long as inverse demand is concave.*

Proof. See Appendix A.1. □

2.2 Firm values with internalization

We finish our overview of the model by deriving firm value expressions under (2). These payoffs do not involve any particular mathematical difficulty, but are not entirely intuitive either and so we detail their construction.

The first step is to state the private values for the firms, i.e. the values of cash flows without internalization. We number firms in the order of their entry and let $\tau_i, i \in \{1, 2, 3\}$ denote a given set of entry times. Denote the state at a given time t by $Y(t) = Y$. For the third firm,

$$V_3^0(Y) = \mathbb{E}_Y \left[\int_{\max\{t, \tau_3\}}^{\infty} D_3(\lambda) Y(u) e^{-r(u-t)} du - \mathbf{1}_{\tau_3 \geq t} I e^{-r(\tau_3-t)} \right] \quad (5)$$

where $\mathbf{1}_A$ is the indicator function, which takes the value 1 if condition A is true and zero otherwise. The payoff reflects that at some τ_3 the investment cost is paid and the corresponding profit flow starts, but if $t > \tau_3$ so the investment has occurred, the investment cost is sunk and firm value captures only decision-relevant cash flows.

Similarly for the second firm,

$$\begin{aligned} V_2^0(Y) = \mathbb{E}_Y & \left[\mathbf{1}_{t < \tau_3} \int_{\max\{t, \tau_2\}}^{\tau_3} D_2(\lambda) Y(u) e^{-r(u-t)} du \right. \\ & \left. \int_{\max\{t, \tau_3\}}^{\infty} D_3(\lambda) Y(u) e^{-r(u-t)} du - \mathbf{1}_{\tau_2 \geq t} I e^{-r(\tau_2-t)} \right], \end{aligned} \quad (6)$$

so with respect to $V_3^0(Y)$ the entrance of the third firm at τ_3 is captured by a reduction in the profit flow from $D_2(\lambda)$ to $D_3(\lambda)$ (if $t < \tau_3$).

For the first firm,

$$\begin{aligned} V_1^0(Y) = \mathbb{E}_Y & \left[\mathbf{1}_{t < \tau_2} \int_{\max\{t, \tau_1\}}^{\tau_2} D_1 Y(u) e^{-r(u-t)} du + \mathbf{1}_{t < \tau_3} \int_{\max\{t, \tau_2\}}^{\tau_3} D_2(\lambda) Y(u) e^{-r(u-t)} du \right. \\ & \left. \int_{\max\{t, \tau_3\}}^{\infty} D_3(\lambda) Y(u) e^{-r(u-t)} du - \mathbf{1}_{\tau_1 \geq t} I e^{-r(\tau_1-t)} \right]. \end{aligned} \quad (7)$$

Perceived firm values are obtained by applying (2) to $V_1^0(Y)$, $V_2^0(Y)$, and $V_3^0(Y)$. The following are some key expressions in our analysis.

At any time $t \in (\tau_2, \tau_3)$ where the first two firms have entered but the third has not, the perceived value of the third firm is

$$V_3^\lambda(Y) = \mathbb{E}_Y \left[\int_t^{\tau_3} 2\lambda D_2(\lambda) Y(u) e^{-r(u-t)} du + \int_{\tau_3}^{\infty} (1 + 2\lambda) D_3(\lambda) Y(u) e^{-r(u-t)} du - I e^{-r(\tau_3-t)} \right]. \quad (8)$$

The first integral captures internalized flows associated with the operations of the first two firms who, contrary to the third firm, are already active. The second integral captures again

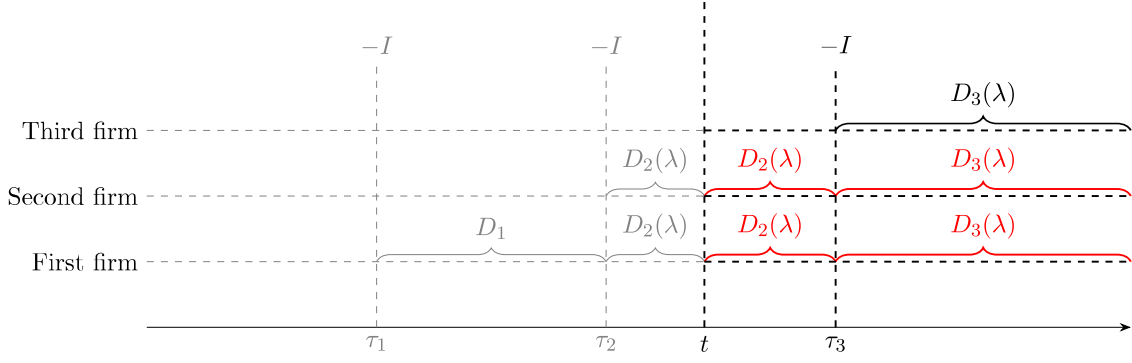


Figure 1: Third firm value with internalization before entering and after the entry of the second firm. Gray represents the past, black represents the firm's own cash flows and red represents the internalized cash flows of other firms.

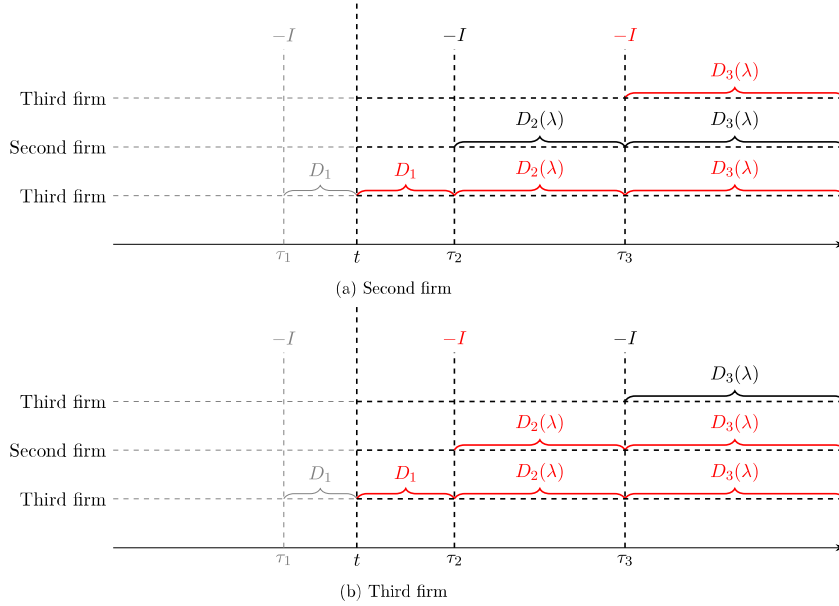


Figure 2: Second and third firm values with internalization with the first firm operating and before either has entered. In all panels, gray represents the past, black represents the firm's own cash flows and red represents the internalized cash flows and investment costs of other firms.

internalized flows of the competitors as well as own flow from the time of the third firm's investment onward. The last term is the third firm's investment cost upon entry, which is paid at $t = \tau_3$. Figure 1 illustrates the relevant cash flows. The expression reduces to $V_3^0(Y)$ if $\lambda = 0$ as expected.

Two other important expressions in our analysis are the values of the two last firms at any time $t \in (\tau_1, \tau_2)$ where the first firm is operating but neither the second nor the third have yet entered. Panel (2a) illustrates the cash flows from the perspective of the second firm. Then the value for the second firm is

$$\begin{aligned}
 V_2^\lambda(Y) = & \mathbb{E}_Y \left[\int_t^{\tau_2} \lambda D_1 Y(u) e^{-r(u-t)} du + \int_{\tau_2}^{\tau_3} (1 + \lambda) D_2(\lambda) Y(u) e^{-r(u-t)} du \right. \\
 & \left. + \int_{\tau_3}^{\infty} (1 + 2\lambda) D_3(\lambda) Y(u) e^{-r(u-t)} du - I e^{-r(\tau_2-t)} - \lambda I e^{-r(\tau_3-t)} \right], \quad (9)
 \end{aligned}$$

where the first integral reflects internalized flows associated with the operations of the active

first firm, the second summand captures again internalized flows of the competitor and own flow from the time of investment, and the third integral captures own and internalized flows upon the entry of the third firm. The last two terms are the own investment cost and the internalized investment cost of the third firm respectively. Again, as expected when $\lambda = 0$ we obtain $V_2^0(Y)$.

The cash flows of the third firm in this same example are illustrated in panel (2b). The value of the third firm at such $t \in (\tau_1, \tau_2)$ is

$$\begin{aligned} V_3^\lambda(Y) = & \mathbb{E}_Y \left[\int_t^{\tau_2} \lambda D_1 Y(u) e^{-r(u-t)} du + \int_{\tau_2}^{\tau_3} 2\lambda D_2(\lambda) Y(u) e^{-r(u-t)} du \right. \\ & \left. + \int_{\tau_3}^{\infty} (1 + 2\lambda) D_3(\lambda) Y(u) e^{-r(u-t)} du - \lambda I e^{-r(\tau_2-t)} - I e^{-r(\tau_3-t)} \right], \quad (10) \end{aligned}$$

Comparing with (9), the differences between the payoffs reside in the duopoly profit flow on the one hand, which is entirely internalized by the third firm so the coefficient is 2λ rather than $1 + \lambda$, and in the investment costs on the other hand where the internalization is reversed, i.e. the third firm internalizes the second firm's investment cost and pays the entire third investment.

Another comparison is with the third firm payoff (8). By moving the time at which the payoff is evaluated to $t \in (\tau_1, \tau_2)$ instead of $t \in (\tau_2, \tau_3)$, the third firm internalizes the entire period of duopoly profit flows as well as part of the monopoly period, and also accounts for the second firm's investment cost which is not yet sunk if $t < \tau_2$. Similar principles apply to the evaluation of the first and second firm payoffs which arise in our equilibrium analysis.

3 Equilibrium

We next determine the equilibrium pattern of entries in the model. In the absence of common ownership, the only equilibrium if firms are symmetric involves all firms entering sequentially (Matros et al., 2024). Introducing internalization enriches the set of equilibrium outcomes significantly, through a variety of different mechanisms. Because firms trivially maximize joint value resulting in a single entry at the monopoly threshold if $\lambda = 1$, we assume hereafter $\lambda < 1$ to focus on the more novel cases.

Optimal investment decisions with common ownership are based on perceived profit incentives rather than private cash flows, i.e. $\Delta_i(\lambda)$ s rather than $D_i(\lambda)$ s. This matters for equilibrium because the last remaining firm is free to time its entry optimally, and internalization can have opposing effects on investment incentives as compared with private cash flows. Introducing common ownership most often lowers the last firm's investment incentive, resulting in delayed entry and making earlier entry more attractive for preceding firms. An extreme case is if the third firm's investment incentive takes negative values, as may occur for some intermediate internalization levels. In such situations the third firm stays out of the market, but still exerts an influence on outcomes by putting competitive pressure on those firms that do enter.

With three firms it is the duopoly stage, where the last two remaining firms compete, that plays the central role in determining the equilibrium. A familiar mechanism in the literature is that sufficiently strong competition between the last two firms pushes the entry trigger for the second industry slot low enough that first-stage competition is eliminated. In such situations, the first two investments occur simultaneously and are said to be clustered. With common ownership this mechanism becomes sufficiently strong to operate, where it otherwise would not

under the assumptions of our model.

In addition to this familiar effect on the first two entries, common ownership introduces another mechanism at the duopoly stage. This mechanism stems from the internalization of the cash flows of the first firm by the remaining two. These internalized cash flows represent a form of dividend for remaining firms, making them more patient or willing to wait. It follows that the two firms may be able to coordinate their investments and forgo racing to preempt one another, resulting in a joint exercise of their expansion options.

To determine the equilibrium in the presence of these different forces, we proceed by backward induction from the last firm up to the initial three-firm preemption stage. We therefore start by studying the investment decision of the third investor, showing that internalization generally softens its entry incentive. We then study the equilibrium timing of the second investor, showing that there is either more intense competition at this stage due to the third investor's softer entry decision or coordination between the last two investors. Finally, we study the initial entry stage where the firms engage in a three-way preemption race.

3.1 Third firm

Once the other two firms have entered, the last firm's investment decision is no longer strategic. Its entry decision therefore amounts to exercising a standard growth option. The third firm's value is

$$F_3(Y) = \sup_{\tau_3 \geq 0} \mathbb{E}_Y \left[\int_0^{\tau_3} 2\lambda D_2(\lambda) Y(t) e^{-rt} dt + \int_{\tau_3}^{\infty} (1 + 2\lambda) D_3(\lambda) Y(t) e^{-rt} dt - I e^{-r\tau_3} \right], \quad (11)$$

where τ_3 is a stopping time and we use the notation $F(\cdot)$ to emphasize this is an option rather than an instantaneous investment value. The term $2\lambda D_2(\lambda) Y(t)$ in the first integral term accounts for the third firm's perceived profit flow while it is inactive and the first two firms operate as duopolists. Upon its entry at τ_3 , the third firm's perceived profit flow shifts to $(1 + 2\lambda) D_3(\lambda) Y(t)$ at cost I . The net change in baseline profit from investment is therefore $\Delta_3(\lambda) = (1 + 2\lambda) D_3(\lambda) - 2\lambda D_2(\lambda)$. Provided that the net change is positive, the third firm invests at the finite threshold

$$Y_3^*(\lambda) = \frac{\beta}{\beta - 1} \frac{r - \mu}{\Delta_3(\lambda)} I \quad (12)$$

where $\beta > 1$ is the positive root of $\frac{1}{2}\sigma^2 x(x - 1) + \mu x - r = 0$. The third firm's value in this case is

$$F_3(Y) = \begin{cases} \frac{2\lambda D_2(\lambda)}{r - \mu} Y + \frac{I}{\beta - 1} \left(\frac{Y}{Y_3^*(\lambda)} \right)^\beta & Y < Y_3^*(\lambda) \\ \frac{(1 + 2\lambda) D_3(\lambda)}{r - \mu} Y - I & Y \geq Y_3^*(\lambda) \end{cases} \quad (13)$$

(see Appendix A.2). If investment does not increase baseline profit, i.e. if $\Delta_3(\lambda) \leq 0$, the option component is worthless. In this case the third firm never exercises its option and its value is simply $F_3(Y) = \frac{2\lambda D_2(\lambda)}{r - \mu} Y$.

3.2 Duopoly subgame

Once the first firm has entered, the remaining two firms engage in a preemption race that determines the entry timing for the second firm, given the anticipated entry decision of the third

firm. The last two firms perceive a positive profit flow from the first firm's monopoly operation due to internalization in our model even while inactive. Because of this perceived profit flow two kinds of equilibrium may arise involving either preemptive or simultaneous investment. We examine the two kinds of equilibrium successively.

3.2.1 Preemptive equilibrium

Start with the case where the third firm has a finite investment threshold ($\Delta_3(\lambda) > 0$). Given the third firm's threshold policy, investing immediately as the second firm yields the instantaneous payoff

$$V_2^\lambda(Y) = \mathbb{E}_Y \left[\int_0^{\tau_3^*} (1 + \lambda) D_2(\lambda) Y(t) e^{-rt} dt + \int_{\tau_3^*}^{\infty} (1 + 2\lambda) D_3(\lambda) Y(t) e^{-rt} dt - I(1 + \lambda e^{-r\tau_3^*}) \right], \quad (14)$$

where $\tau_3^* = \inf\{t \geq 0 : Y(t) \geq Y_3^*(\lambda)\}$ is the third firm's anticipated entry time. Entry gives the second firm access to the perceived duopoly profit flow in the first integral term, $(1 + \lambda) D_2(\lambda) Y(t)$, at immediate cost I . The third firm then enters at τ_3^* , transforming the market into a triopoly. The perceived profit flow shifts to $(1 + 2\lambda) D_3(\lambda) Y(t)$ thereafter, and the second firm internalizes part of the third firm's investment cost.

The instantaneous value of following, i.e. of remaining inactive as the other firm enters and eventually entering as the third firm, is

$$F_3(Y) - \lambda I = \mathbb{E}_Y \left[\int_0^{\tau_3^*} 2\lambda D_2(\lambda) Y(t) e^{-rt} dt + \int_{\tau_3^*}^{\infty} (1 + 2\lambda) D_3(\lambda) Y(t) e^{-rt} dt - I(\lambda + e^{-r\tau_3^*}) \right]. \quad (15)$$

$F_3(Y)$ is introduced above, so the key point here is that because the firms are comparing the payoffs of leading versus following at the preemption stage, the follower payoff incorporates the internalized leader entry cost $-\lambda I$ which has not yet been sunk (see Figure 2 for an illustration of cash flows).

The difference between $V_2^\lambda(Y)$ and $F_3(Y) - \lambda I$ is the preemption incentive. In a sequential equilibrium the entry of the second firm is triggered at the preemption threshold, denoted $Y_2^P(\lambda)$, which is the lower bound of the demand shock levels where the preemption incentive is positive. Both firms seek to enter at this threshold, and we suppose for simplicity that either firm is equally likely to be the second entrant.³

If $\Delta_3(\lambda) \leq 0$ so the third firm never chooses to enter, the reasoning is similar but the preemption incentive takes a simpler form. Competition to be the second firm is then at its most intense, with entry occurring at the break-even threshold $\bar{Y}_2(\lambda)$ which represents the lower bound of possible preemption thresholds.

Proposition 2 (Duopoly stage preemption). *In a preemptive equilibrium:*

(i) *If $\Delta_3(\lambda) > 0$ then the second firm's entry trigger $Y_2^P(\lambda) < Y_3^*(\lambda)$ is implicitly defined by*

$$\left(Y_2^P(\lambda) - \frac{(r - \mu) I}{D_2(\lambda)} \right) (Y_2^P(\lambda))^{-\beta} = \left(Y_3^*(\lambda) - \frac{(r - \mu) I}{D_2(\lambda)} \right) (Y_3^*(\lambda))^{-\beta}. \quad (16)$$

³See (Thijssen, Huisman, & Kort, 2012) for a rigorous presentation of equilibrium strategies in a symmetric preemption game under uncertainty.

(ii) If $\Delta_3(\lambda) \leq 0$ then the second firm's entry trigger is $\bar{Y}_2(\lambda) = \frac{r-\mu}{D_2(\lambda)}I$.

Proof. See Appendix A.3. □

3.2.2 Simultaneous investment equilibrium

Unlike with generic new market entry, the second and third firms in our model perceive positive profit flows from the first firm even while inactive. The value of waiting and coordinating investments may therefore dominate the value of racing. Another kind of equilibrium can therefore arise where the two remaining firms invest simultaneously.⁴

Suppose that instead of competing to enter first, the second and third firms opt to exercise their investment options jointly. This joint exercise policy yields value

$$F_2^S(Y) = \sup_{\tau_2^S \geq 0} \mathbb{E}_Y \left[\int_0^{\tau_2^S} \lambda D_1 Y(t) e^{-rt} dt + \int_{\tau_2^S}^{\infty} (1+2\lambda) D_3(\lambda) Y(t) e^{-rt} dt - (1+\lambda) I e^{-r\tau_2^S} \right]. \quad (17)$$

The first integral term above captures the perceived profit flow from the first firm, λD_1 , while the two remaining firms are inactive. The two remaining firms enter simultaneously at τ_2^S , shifting the profit flow directly to the three-firm level $(1+2\lambda) D_3(\lambda) Y(t)$, at a perceived cost from joint investment $(1+\lambda)I$. The net change in baseline profit from investment is $\Delta_2^S(\lambda) = (1+2\lambda) D_3(\lambda) - \lambda D_1$.

A necessary condition for simultaneous investments to eventually occur is that $\Delta_2^S(\lambda)$ be positive, which implies in turn that $\Delta_3(\lambda)$ is positive.⁵ If this is the case, the optimal joint exercise policy involves simultaneous investments at the threshold

$$Y_2^S(\lambda) = \frac{\beta}{\beta-1} \frac{r-\mu}{\Delta_2^S(\lambda)} (1+\lambda) I. \quad (18)$$

The resulting value is

$$F_2^S(Y) = \begin{cases} \frac{\lambda D_1}{r-\mu} Y + \frac{(1+\lambda)I}{\beta-1} \left(\frac{Y}{Y_2^S(\lambda)} \right)^\beta & Y < Y_2^S(\lambda) \\ \frac{(1+2\lambda)D_3(\lambda)}{r-\mu} Y - (1+\lambda) I & Y \geq Y_2^S(\lambda). \end{cases} \quad (19)$$

Otherwise ($\Delta_2^S(\lambda) \leq 0$), in a simultaneous equilibrium the two firms refrain from ever investing and the value from opting for such a policy is simply $F_2^S(Y) = \frac{\lambda D_1}{r-\mu} Y$.

For a simultaneous investment equilibrium to arise, neither firm must have an incentive to preempt the other before the joint investment threshold is reached. This is established by checking that $F_2^S(Y)$ lies above the instantaneous investment payoff $V_2^\lambda(Y)$ over the relevant range $(Y_2^P(\lambda), Y_2^S(\lambda))$. When this is the case there is generally a range of demand states over which simultaneous investment is sustainable. In line with the literature, we suppose that firms coordinate on investing at the Pareto-superior equilibrium, $Y_2^S(\lambda)$, if there are multiple equilibria.

The following proposition states the result.

⁴See (Fudenberg & Tirole, 1985) and (Chevalier-Roignant & Trigeorgis, 2011) for discussions of such equilibria.

⁵Because $\Delta_2^S(\lambda) = \Delta_3(\lambda) - \lambda(D_1 - 2D_2(\lambda))$ and $D_1 \geq 2D_2(\lambda)$ by Assumption A(i), we have $\Delta_2^S(\lambda) \leq \Delta_3(\lambda)$.

Proposition 3 (Duopoly stage coordination). *A simultaneous investment equilibrium exists if $\Delta_3(\lambda) > 0$ and*

$$\left(\beta(1 - \lambda) \left(\frac{D_2(\lambda)}{\Delta_3(\lambda)} - 1 \right) - \lambda \right) \left(\frac{\Delta_3(\lambda)}{\Delta_2(\lambda)} \right)^\beta + \mathbf{1}_{\Delta_2^S > 0} \frac{1}{(1 + \lambda)^{\beta-1}} \left(\frac{\Delta_2^S(\lambda)}{\Delta_2(\lambda)} \right)^\beta \geq 1, \quad (20)$$

in which case the second and third firms invest at the threshold $Y_2^S(\lambda)$ if $\Delta_2^S(\lambda) > 0$ and abstain from investing otherwise.

Proof. See Appendix A.3. □

The condition in the proposition requires that the option value from joint investment (if firms invest at a finite threshold) be larger than the value from investing optimally as the second firm, given the anticipated entry of the last firm.⁶ As firms coordinate on investing at the Pareto-superior equilibrium, if (20) holds so there are both preemptive and simultaneous investment equilibria at the duopoly stage, it follows that the firms opt for the simultaneous investment equilibrium which yields them a higher payoff.

3.3 Initial entry stage

Finally we determine the entry timing for the first investment at the initial stage of the game. At this stage firm roles are undetermined and the firms engage in a three-way preemption race given the subsequent investment behavior of the remaining firms which is described above.

First of all, it may occur that in the duopoly stage there are both preemptive and simultaneous investment equilibria, if condition (20) holds. If this is the case, the assumption that firms coordinate on the Pareto-superior implies that the first firm to enter expects the two remaining firms to play the simultaneous investment equilibrium of Proposition 3. There are two subcases, depending on whether the last two firms invest jointly at a finite threshold or abstain from ever investing. In the former case, the investment policy of the second and third firms implies that the instantaneous payoff from investing first is

$$\hat{V}_1^\lambda(Y) = \mathbb{E}_Y \left[\int_0^{\tau_2^S} D_1 Y(t) e^{-rt} dt + \int_{\tau_2^S}^\infty (1 + 2\lambda) D_3(\lambda) Y(t) e^{-rt} dt - \left(1 + 2\lambda e^{-r\tau_2^S} \right) I \right], \quad (21)$$

where $\tau_2^S = \inf\{t \geq 0 : Y_t \geq Y_2^S(\lambda)\}$ is the anticipated entry time of the last two firms. The value expression above consists of the value of the monopoly profit flow, along with a correction term that reflects the entry of the two remaining firms which switches the perceived profit flow to the three-firm equilibrium and incorporates the perceived investment cost of the remaining firms.

The value from following for the remaining firms is the value of exercising their joint investment option net of the internalized leader investment cost, $F_2^S(Y) - \lambda I$. It can be shown that

⁶Denote the optimal threshold for the second firm by $Y_2^*(\lambda) = \frac{\beta}{\beta-1} \frac{r-\mu}{\Delta_2(\lambda)} I$. Then (20) is equivalent to

$$\mathbf{1}_{\Delta_2^S > 0} \frac{(1 + \lambda)I}{\beta - 1} \left(\frac{Y_2^*(\lambda)}{Y_2^S(\lambda)} \right)^\beta \geq \frac{I}{\beta - 1} + \left(\frac{(1 + 2\lambda)D_3(\lambda) - (1 + \lambda)D_2(\lambda)}{r - \mu} Y_3^*(\lambda) - \lambda I \right) \left(\frac{Y_2^*(\lambda)}{Y_3^*(\lambda)} \right)^\beta.$$

The term on left-hand side of the inequality represents the option value of simultaneous investment. The term on the right-hand side is the option value from investment at the optimal second firm threshold, which is the sum of a perpetual profit flow increase and a correction term accounting for the effect of the third firm's entry.

the preemption incentive is positive over a range of demand states, resulting in an implicitly defined preemption threshold.

In the second subcase, the two remaining firms abstain from ever investing and the preemption incentive takes a simpler form because there is no correction term. First-stage preemption then results in a monopoly outcome, with a single investment that occurs at the breakeven threshold.

We next move to the case where condition (20) does not hold. Then the first firm to enter expects the two remaining firms to play the sequential investment equilibrium of Proposition 2. There are two subcases again, depending on whether the last firm enters or not. If it does, i.e. if the third firm threshold is finite, then the first firm expects remaining investments to take place at $Y_2^P(\lambda)$ and $Y_3^*(\lambda)$ and investing first yields an instantaneous payoff

$$\begin{aligned} V_1^\lambda(Y) = & \mathbb{E}_Y \left[\int_0^{\tau_2^P} D_1 Y(t) e^{-rt} dt + \int_{\tau_2^P}^{\tau_3^*} (1 + \lambda) D_2(\lambda) Y(t) e^{-rt} dt + \right. \\ & \left. + \int_{\tau_3^*}^\infty (1 + 2\lambda) D_3(\lambda) Y(t) e^{-rt} dt - \left(1 + \lambda e^{-r\tau_2^P} + \lambda e^{-r\tau_3^*} \right) I \right], \end{aligned} \quad (22)$$

where $\tau_2^P = \inf\{t \geq 0 : Y_t \geq Y_2^P(\lambda)\}$ and $\tau_3^* = \inf\{t \geq 0 : Y_t \geq Y_3^*(\lambda)\}$ are the anticipated entry times of the second and third firms. The instantaneous value of being second in this case is

$$\begin{aligned} \mathbb{E}_Y \left[\int_0^{\tau_2^P} \lambda D_1 Y(t) e^{-rt} dt + \int_{\tau_2^P}^{\tau_3^*} (1 + \lambda) D_2(\lambda) Y(t) e^{-rt} dt + \right. \\ \left. + \int_{\tau_3^*}^\infty (1 + 2\lambda) D_3(\lambda) Y(t) e^{-rt} dt - \left(\lambda + e^{-r\tau_2^P} + \lambda e^{-r\tau_3^*} \right) I \right], \end{aligned} \quad (23)$$

as the second firm internalizes the first firm's entry cost which has not yet been sunk. The preemption incentive, which is the difference of the above payoffs, turns out not to be necessarily positive. If it is, then a preemption race takes place for the first entry slot resulting in an outcome where all three firms enter sequentially. But if the preemption incentive is not positive for any demand shock, another equilibrium outcome occurs where the two first firms enter simultaneously at the preemption threshold of the second firm.

In the second subcase, the third firm abstains from ever investing which simplifies the payoff expressions, but the reasoning is otherwise similar. The resulting equilibrium outcome then involves just two firms which invest either sequentially at their respective preemption thresholds or simultaneously at the preemption threshold of the second firm.

In either of these subcases with sequential equilibrium at the duopoly stage, the triggers of the first two firms coincide resulting in clustering of investments if the preemption threshold of the second firm is lower than the monopoly threshold of the first firm, as there is then no unilateral incentive for a firm to enter at a lower demand state. The condition for such a situation to arise is

$$D_2(\lambda) - \frac{\beta - 1}{\beta} D_1 \frac{1 - \left(\frac{\max\{\Delta_3(\lambda), 0\}}{D_1} \right)^\beta}{1 - \left(\frac{\max\{\Delta_3(\lambda), 0\}}{D_1} \right)^{\beta-1}} \geq 0. \quad (24)$$

The following proposition summarizes our results:

Proposition 4 (Three-firm equilibrium). *In an equilibrium of the entry game with low enough initial state Y :*

(i) (Coordination) *If (20) holds, then either the first entry trigger $\hat{Y}_1^P(\lambda) < Y_2^S(\lambda)$ is implicitly defined by*

$$\left(\hat{Y}_1^P(\lambda) - \frac{(r-\mu)I}{D_1} \right) \left(\hat{Y}_1^P(\lambda) \right)^{-\beta} = \left(Y_2^S(\lambda) - \frac{(r-\mu)I}{D_1} \right) (Y_2^S(\lambda))^{-\beta}, \quad (25)$$

and the two remaining firms enter at $Y_2^S(\lambda)$ (if $\Delta_2^S(\lambda) > 0$), or the first firm enters at the breakeven threshold $\bar{Y}_1 = \frac{(r-\mu)I}{D_1}$ and the two remaining firms never enter (if $\Delta_2^S(\lambda) \leq 0$).

(ii) (Clustering) *If (20) does not hold but (24) does hold, then either the first two firms enter at the duopoly trigger $Y_2^P(\lambda)$ and the third firm enters at $Y_3^*(\lambda)$ (if $\Delta_3(\lambda) > 0$), or the first two firms enter at the breakeven duopoly threshold $\bar{Y}_2(\lambda)$ and the third firm never enters (if $\Delta_3(\lambda) \leq 0$).*

(iii) (Sequential) *If neither (20) nor (24) hold, then the first entry trigger $Y_1^P(\lambda) < Y_2^P(\lambda)$ is implicitly defined by*

$$\left(Y_1^P(\lambda) - \frac{(r-\mu)I}{D_1} \right) (Y_1^P(\lambda))^{-\beta} = \left(Y_2^P(\lambda) - \frac{(r-\mu)I}{D_1} \right) (Y_2^P(\lambda))^{-\beta}, \quad (26)$$

and either the second firm enters at the duopoly trigger $Y_2^P(\lambda)$ and the third firm enters at $Y_3^(\lambda)$ (if $\Delta_3(\lambda) > 0$), or the second firm enters at the breakeven duopoly threshold $\bar{Y}_2(\lambda)$ and the third firm never enters (if $\Delta_3(\lambda) \leq 0$).*

Proof. See Appendix A.4. □

3.4 Discussion

Our equilibrium result has implications for how industries develop, when products are introduced on the market, and how entry timing reacts to changes in common ownership which we now turn to. To work out the implications, Figure 3 illustrates the different cases of Proposition 4 and corresponding outcomes.

Of the different outcomes, two are familiar from the entry timing literature. If the third firm enters and second stage competition is not too intense, the *Full Sequential* equilibrium arises where all firms enter sequentially at the preemption triggers Y_1^P and Y_2^P and optimal threshold Y_3^* respectively. If the third firm enters and second stage competition is sufficiently intense with firms still preferring to race, the *Clustering* equilibrium arises with the first two firms entering at the preemptive trigger Y_2^P and the third at Y_3^* .

Other outcomes are specific to our model. First, it may be that the third firm has a negative entry incentive due to internalization. If so, the Full Sequential equilibrium becomes a *Duopoly Sequential* equilibrium involving an all the more aggressive preemption race as the three firms effectively vie for two slots, and entries occur at the preemptive trigger Y_1^P and breakeven duopoly threshold $\bar{Y}_2$. Similarly, Clustering becomes a *Duopoly Contest* where two of the three firms enter simultaneously at the breakeven duopoly threshold.

Second, as indicated by the leftmost branch it can also occur that the remaining firms coordinate at the duopoly stage. If the optimal joint exercise involves both firms entering at a finite threshold, there is a *Follower Coordination* equilibrium with initial investment taking

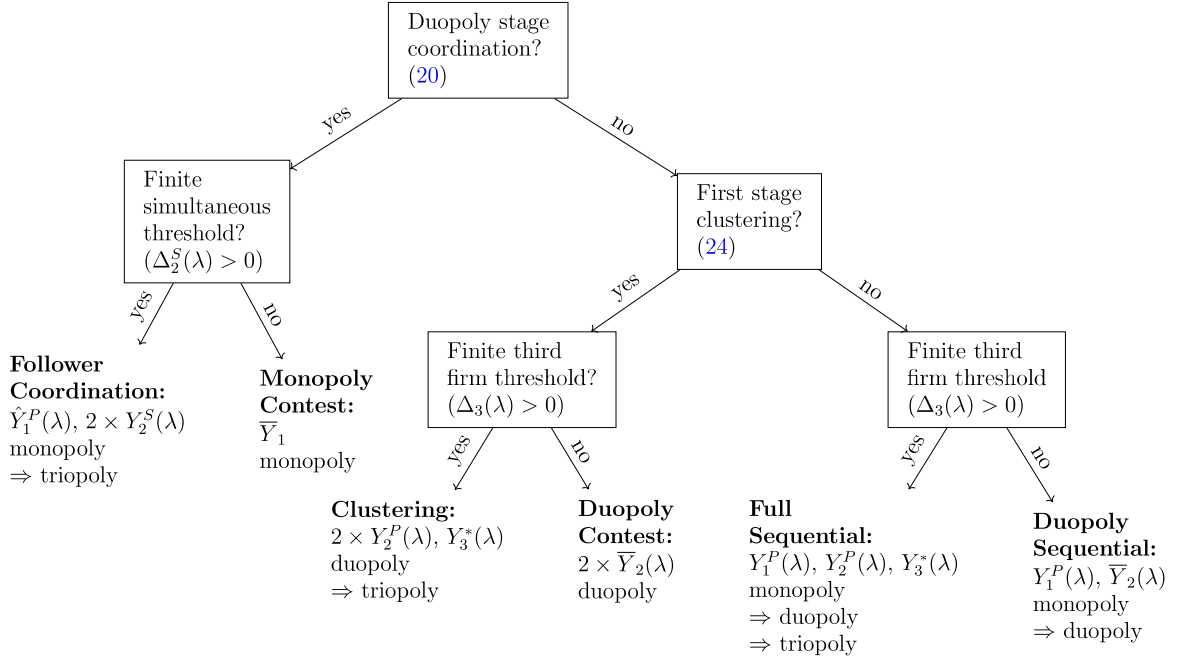


Figure 3: Equilibrium outcomes and industry development in Proposition 4 ($2\times$ denotes two firms investing simultaneously and $\Rightarrow$ the progression of market structures starting from an initially empty market).

place at a more aggressive preemption trigger $\hat{Y}_1^P$ and the two remaining entries at an optimal threshold Y_2^S . The optimal coordination may also involve never investing at all, in which case the three firms compete for a single slot in a *Monopoly Contest* equilibrium where a single entry occurs at the breakeven threshold $\bar{Y}_1$.

Implications for industry structure and development A first observation is that common ownership can lead the industry to operate with fewer than three firms, in several equilibria. When this happens, the presence of potential competition lowers entry triggers to competitive levels. In such equilibria, one or two firms may remain inactive even though entry is privately profitable. Conversely, some firms may invest despite perceiving a negative incentive, in order to avoid ending up as the last inactive firm (e.g. if $\Delta_2(\lambda) \leq 0$ and the second stage equilibrium is sequential, then $\Delta_3(\lambda) \leq 0$ implying that the second firm invests at $\bar{Y}_2(\lambda)$ even though this lowers perceived cash flows).

Second, the industry can evolve towards its ultimate configuration either gradually, i.e. through a sequence of entries, or through “investment waves”, i.e. jumps in the number of active firms. Investment waves may themselves result from different underlying mechanisms, pertaining to excessive or insufficient competition at different stages. On the one hand, intense competition at the duopoly stage may neutralize initial competition, causing the industry to transition directly to duopoly (Clustering equilibrium). On the other hand, if firms coordinate investments at the duopoly stage the industry can jump from one firm to three (Coordination equilibrium), which paradoxically exacerbates competition to enter the industry first. In the extreme case where the last two firms abstain from ever investing, the market is served by a single monopolist entering at its breakeven threshold, i.e. a winner-take-all market whose rents

are entirely dissipated.

First firm entry Internalization accelerates some investments while delaying others, so the normative properties of equilibrium are generally complex. A simple metric to gauge the strength of dynamic competition is the first firm's entry threshold. A first motivation for using this measure is that it is initial market entry that sparks market activity, and moreover for standard demand forms the first entry usually represents the most significant welfare impact for consumers.

A benchmark result in the literature is that the first entry trigger in a three-firm industry lies between corresponding two-firm and monopoly levels, meaning that the addition of a third firm somewhat counterintuitively weakens overall competition rather than intensifying it. If the equilibrium is of the sequential type, a similar logic continues to hold with internalization: the two firms remaining after the first entry still compete with one another, weakening the incentive to enter first as compared with a two-firm industry and thereby delaying first firm entry.

If the equilibrium at the duopoly stage is of the coordination type however, the comparison of two and three-firm industries is reversed. Internalization softens competition between the last two firms drastically in this case, prompting the two remaining firms to enter at an even higher threshold than a follower in a two-firm industry. Competition to be the first firm on the market is therefore extremely intense, resulting in earlier first entry than in a two-firm industry and restoring the intuition that increasing firm numbers should intensify overall competition. The effect is most pronounced if the last two firms abstain from investment entirely in the simultaneous equilibrium, as competition to enter first becomes a race for the market which drives the first firm's entry to its lowest possible level, namely the monopoly break-even threshold $\bar{Y}_1$.

If the equilibrium is of the clustering type finally, first entry still occurs earlier than the monopoly threshold but the comparison with a two-firm industry is indeterminate. Formally, and denoting with $Y_1^{(2)P}(\lambda)$ the preemption threshold with two firms,⁷ we have:

Proposition 5 (First entry comparison). *Compared with the first entry thresholds Y_1^* in an one-firm industry and $Y_1^{(2)P}(\lambda)$ in a two-firm industry, the first entry threshold in a three-firm industry lies:*

- (i) *between these two, i.e. $Y_1^P(\lambda) \in (Y_1^{(2)P}(\lambda), Y_1^*)$, if the equilibrium is of the Sequential type;*
- (ii) *below them, i.e. $\hat{Y}_1^P(\lambda) < Y_1^{(2)P}(\lambda) < Y_1^*$, if the equilibrium is of the Coordination type.*

Proof. See Appendix A.5. □

Comparative statics with respect to internalization We next turn to the sensitivity of equilibrium and entry timing to internalization. With respect to equilibrium type, the derivatives of (20) are too complex to sign analytically so we defer further discussion to the next section. On the other hand, condition (24) which determines whether the investments of the first two firms cluster can be characterized analytically. The key insight is that competition between the last

⁷The preemption threshold in a two-firm industry is the lower root of

$$\left(Y_1^{(2)P}(\lambda) - \frac{(r-\mu)I}{D_1} \right) (Y_1^{(2)P}(\lambda))^{-\beta} = \left(Y_2^*(\lambda) - \frac{(r-\mu)I}{D_1} \right) (Y_2^*(\lambda))^{-\beta},$$

where $Y_2^*(\lambda) = \frac{\beta}{\beta-1} \frac{r-\mu}{\Delta_2(\lambda)} I$ is the follower threshold under duopoly.

two firms is more intense because of the last firm's internalization, and this effect of subsequent competition can be strong enough that the investment triggers of the first and second firms coincide. The following proposition states the result.

Proposition 6 (Internalization and clustering). *Provided Assumption C holds and the equilibrium of the duopoly stage is of the sequential type, an increase in internalization raises the likelihood that the first two firms invest simultaneously at $Y_2^P(\lambda)$.*

Proof. See Appendix A.6. □

The other, and perhaps most important sensitivity result concerns the timing of equilibrium investments. (Bouis et al., 2009) show that an accordion effect arises when a single flow payoff changes (e.g. either a $\Delta_2(\lambda)$ or a $\Delta_3(\lambda)$), while changes in other model parameters exert the same qualitative effect on all investment thresholds (e.g. an increase in unit cost or volatility raises all thresholds). In our model, internalization is a central parameter whose role is interesting to examine both from a theoretical standpoint but also to gain insights as to the effect of the gradual rise in common ownership brought about by real-world evolutions in institutional investing. The following proposition lays out the effects of internalization on investment timing in our model:

Proposition 7 (Accordion effect). *Provided Assumption C holds the triopoly equilibrium exhibits an accordion effect: for any successive investment triggers Y and Y' , the effects of increased internalization alternate in sign, i.e.*

$$\frac{dY}{d\lambda} \times \frac{dY'}{d\lambda} \leq 0. \quad (27)$$

Proof. See Appendix A.7. □

Because firms in our model may invest simultaneously or abstain from investing in equilibrium, in some cases the accordion effect reduces alternating sign with respect to just two firms. This is the case in the Sequential Duopoly equilibrium, i.e. if neither (20) nor (24) hold and $\Delta_3(\lambda) \leq 0$, in which case the investment thresholds satisfy $\frac{dY_1^P}{d\lambda} > 0$ and $\frac{d\bar{Y}_2}{d\lambda} < 0$. Similarly, in the Clustering equilibrium, i.e. if (24) holds, (20) does not, and $\Delta_3(\lambda) > 0$, we have $\frac{dY_2^P}{d\lambda} < 0$ and $\frac{dY_3^*}{d\lambda} > 0$. The accordion effect applies here not to individual firm thresholds, but to the distinct thresholds at which investments occur. Similarly in the Follower Coordination equilibrium, i.e. if (20) holds and $\Delta_2^S(\lambda) > 0$ in which case $\frac{d\hat{Y}_1^P}{d\lambda} \times \frac{dY_2^S}{d\lambda} \leq 0$ (the sign of $\frac{dY_2^S}{d\lambda}$ is generally indeterminate, as we illustrate in the next section).

A “true” accordion effect arises in the Full Sequential equilibrium, where the threshold effects alternate for all three firms. That is, if neither (20) nor (24) hold and $\Delta_3(\lambda) > 0$ so three firms invest. Then $\frac{dY_1^P}{d\lambda} > 0$, $\frac{dY_2^P}{d\lambda} < 0$, and $\frac{dY_3^*}{d\lambda} > 0$.

Intuitively, the accordion effect arises for changes in the internalization levels because, unlike in the case of a positive profit shock, the third firm's investment depends on the investment incentive $\Delta_3(\lambda)$ which decreases with internalization even if the actual flow profits $D_2(\lambda)$ and $D_3(\lambda)$ increase, and the result also relies on the fact that the monopoly profit D_1 is independent of the internalization level.

We therefore expect rising common ownership to have heterogeneous effects on investment timing, which are better explored with a numerical analysis in the next section.

Linear demand ($P(Q) = a - bQ$, $a > c \geq 0$, $b > 0$)	
Total output	$Q_N(\lambda) = \frac{N}{N+1+\lambda(N-1)} \frac{a-c}{b}$
Profit	$D_N(\lambda) = \frac{1+\lambda(N-1)}{(N+1+\lambda(N-1))^2} \frac{(a-c)^2}{b}$
Isoelastic demand ($P(Q) = AQ^{-\eta}$, $A > 0$, $\eta \in (0, 1)$)	
Total output	$Q_N(\lambda) = \left(\frac{N-\eta(1+\lambda(N-1))}{N} \frac{A}{c} \right)^{\frac{1}{\eta}}$
Profit	$D_N(\lambda) = \frac{\eta c(1+\lambda(N-1))(N-\eta(1+\lambda(N-1)))^{\frac{1-\eta}{\eta}}}{N^{\frac{1+\eta}{\eta}}} \left(\frac{A}{c} \right)^{\frac{1}{\eta}}$

Table 1: Cournot equilibrium with symmetric internalization

4 Equilibrium and entry dynamics in a Cournot market

We further our analysis by positing Cournot competition in the product market stage and specifying demand. We suppose that products are homogeneous and that unit costs are constant and equal to c . For most of the section we use a linear demand specification, but we also examine an isoelastic specification in the last paragraph. Both demand specifications yield product market outcomes that satisfy the assumptions of the model, including Assumption C. Table 1 lists the equilibrium outputs and private profits for both specifications (see Appendix A.1 for product market equilibrium derivation).

We first study the equilibrium and its properties with the linear specification extensively, so as to highlight the roles of option value and internalization in determining equilibrium behavior, entry dynamics, and normative outcomes. Our analysis shows how higher option value and internalization shift the pattern of industry dynamics from one of sequential entries to contest for leadership. This shift towards more contest-like dynamics has significant consequences for business dynamism and welfare, which we show can both increase in contrast with the habitual pattern under sequential entries.

The last part of the section characterizes equilibrium outcomes with isoelastic demand, which allows us to both check the robustness of the linear specification and to highlight the role of demand curvature in equilibrium determination.

4.1 Linear demand

We start by studying the equilibrium if the product market is described by the classical quantity competition model with linear demand. Without loss of generality, we set $a = b = 1$ and $c = 0$. In this case the investment incentives $\Delta_2^S(\lambda)$ and $\Delta_3(\lambda)$ are positive and decreasing for all $\lambda < 1$ by Proposition 1. Then Proposition 4 implies that all three firms eventually must enter the market, so there are just three possible equilibria: Full Sequential, Clustering, and Follower Coordination.

We want to understand first how internalization and option value influence equilibrium outcomes. To this effect, Figure 4 plots the relevant boundaries (20) and (24) in the (λ, β) parameter space. In the baseline case of no internalization ($\lambda = 0$), the only possible equilibrium is Full Sequential. When internalization enters into the picture, Clustering and Follower Coordination

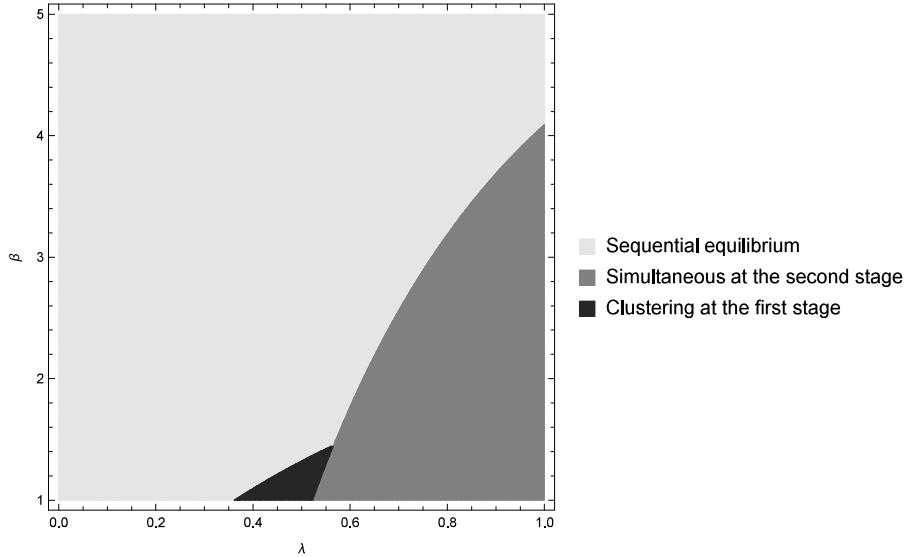


Figure 4: Equilibrium regions with linear demand: Full Sequential (light gray), Clustering (black), and Follower Coordination (dark gray). Firms respectively enter at Y_1^P , Y_2^P , and Y_3^* (Full Sequential), Y_2^P (first two firms) and Y_3^* (Clustering), or $\hat{Y}_1^P$ and Y_2^S (last two firms, Follower Coordination).

equilibria arise depending on the value of β . Intuitively, lower β values make the joint investment option more valuable, which drives firms towards coordinated investments at the duopoly stage if there is sufficient internalization, resulting in a Follower Coordination equilibrium. In addition to this novel equilibrium outcome, the softened behavior of the last entrant pushes the second entry trigger low enough that the first two entries can cluster if internalization is high enough and β is sufficiently low.

We next look more closely at the timing of investments across different equilibria. To calculate investment thresholds, we fix $\mu = 0.01$, $r = 0.05$ and $I = 1$, and let the volatility parameter σ vary. Because β is inversely related to σ for fixed μ and r , increases in β are associated with lower volatility and vice-versa. Figure 5 plots entry thresholds with respect to the level of internalization in three representative equilibrium configurations. At low uncertainty (5a), the equilibrium is invariably Full Sequential. At intermediate uncertainty (5b), the equilibrium is Full Sequential at low internalization but Follower Coordination also arises at high internalization. At high uncertainty finally (5c) all three equilibrium outcomes (Full Sequential, Clustering, and Follower Coordination) are present. Thresholds vary continuously with respect to internalization throughout, except where the equilibrium shifts to Follower Coordination (indicated by λ^* in panel 5b and λ_2^* in panel 5c). At this shift the first investment threshold jumps down and the last investment threshold jumps up. The Full Sequential equilibrium clearly exhibits an accordion effect (Proposition 7). Other equilibrium outcomes also exhibit opposing monotonicity of the first and last entry thresholds. In addition, the threshold jump where equilibrium shifts to Follower Coordination exhibits the same pattern. We conclude that an accordion effect, whereby softer competition at one stage intensifies competition at the preceding stage and vice-versa, characterizes the effect of internalization on investment timing globally in our model.

In contrast with the accordion effect of internalization changes, volatility changes have a uniform effect on (almost all) thresholds, as illustrated in Figure 6. There are four representative equilibrium configurations, which correspond to different vertical sections of Figure 4. The figure

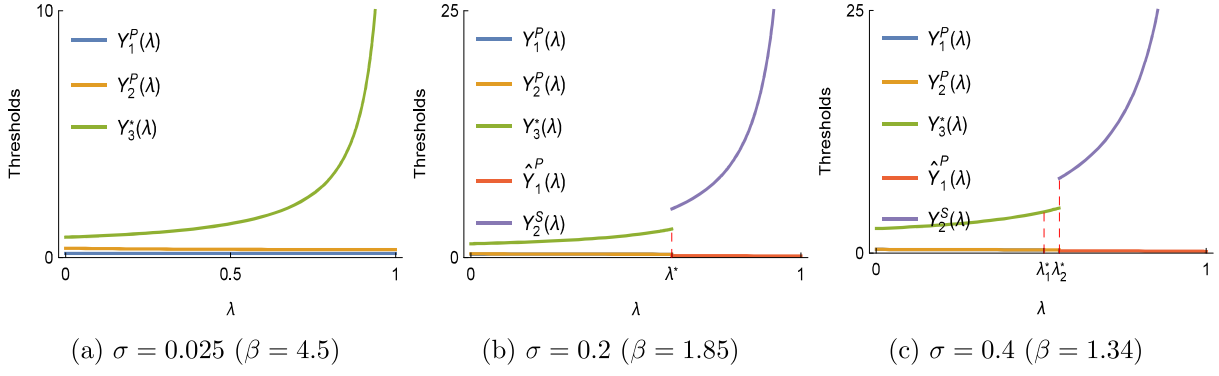


Figure 5: Investment thresholds as functions of λ in linear case. In (5a) equilibrium is invariably Full Sequential. In (5b) Follower Coordination arises for $\lambda \geq \lambda^*$, and thresholds jump. In (5c) the thresholds transition smoothly to a Clustering equilibrium between λ_1^* and λ_2^* , and then jump to Follower Coordination at λ_2^* . Increasing internalization has an accordion effect (Bouis et al., 2009) throughout, as the signs of the effect of internalization on successive thresholds all alternate.

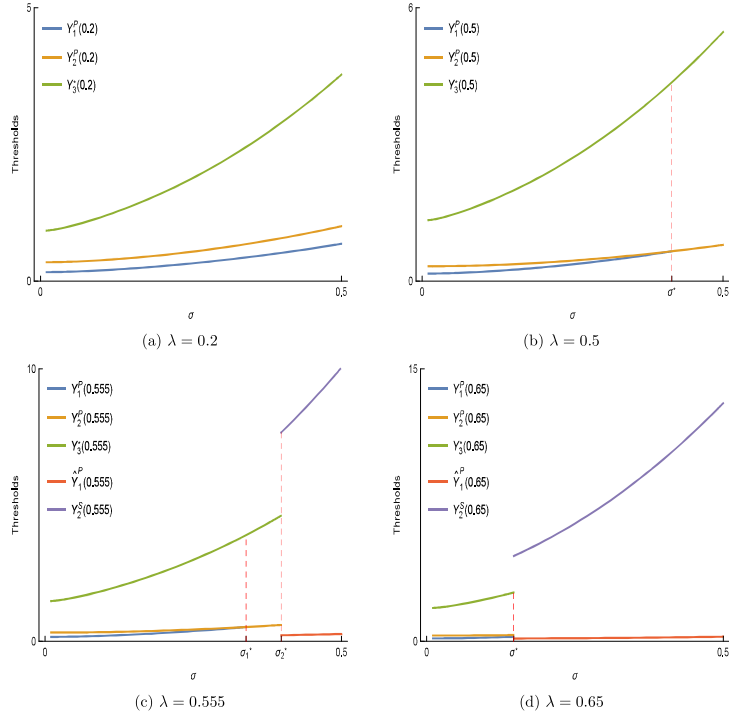


Figure 6: Investment thresholds as a function of σ in linear case. In (6a) only Full Sequential equilibrium applies. In (6b) Clustering occurs for $\sigma \geq \sigma^*$. In (6c) there is Clustering between σ_1^* and σ_2^* , and Duopoly Coordination above σ_2^* . In (6d) there is Duopoly Coordination for $\sigma \geq \sigma^*$.

shows how investment thresholds increase in volatility almost everywhere, in contrast with the accordion effect of internalization. The single exception is the point where equilibrium shifts to Follower Coordination, at which the first investment threshold jumps down while the last investment threshold jumps up, before resuming monotone increasing behavior.

We next examine the entry behavior implied by the threshold functions studied with respect to the decline in business dynamism which has been widely discussed in macroeconomics literature (Akcigit & Ates, 2021). In certain concentrated industries, some evidence also points to increasing entry rates and markups which are more suggestive of contest-like industry dynamics

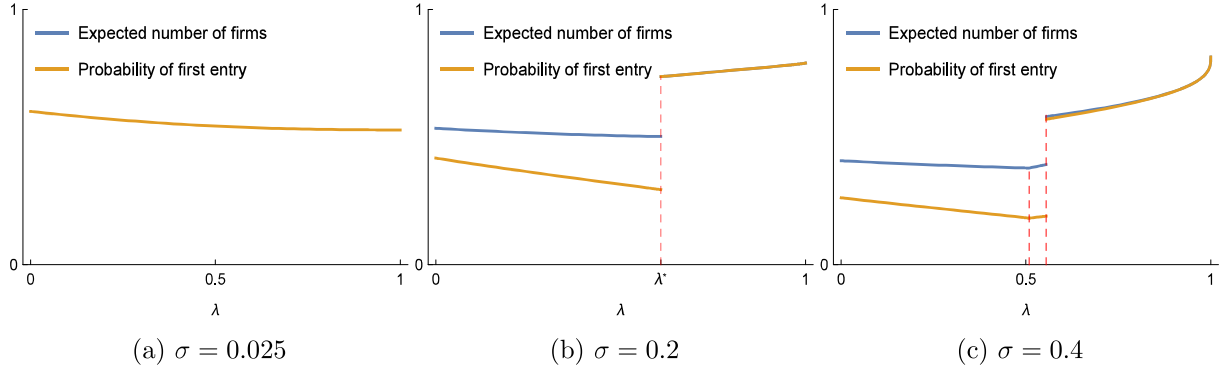


Figure 7: Expected number of firms and probability of a first entry over 25 years as a function of λ and for different volatility levels, starting at $Y = 0.13$. In (7a) expected entries are essentially driven by the first firm and decline with internalization. In (7b), equilibrium jumps to Follower Coordination resulting in higher and increasing expected entries. In (7c), over the range of intermediate internalization levels where the equilibrium is Clustering higher internalization also increases expected entries.

(Albrecht & Decker, 2026). Because our model centers on new market entry in a single industry where no firms are active initially, calculating an entry rate is not directly meaningful. We can however obtain a measure of business dynamism which is more adapted in our setting by using the first hitting probabilities associated with the equilibrium entry thresholds to calculate the expected number of firm entries over a given time period.

We calculate the probability of the i th firm entering by a given time T , $\mathbb{P}(\tau_i \leq T)$, by evaluating the hitting probabilities $\mathbb{P}(\sup_{0 \leq t \leq T} Y(t) \geq Y_i | Y(0) = Y)$, where Y_i denotes the relevant entry trigger or threshold.⁸ Figure 7 plots expected firm entries for different volatility levels, over a time horizon of $T = 25$. At low volatility levels (7a), the equilibrium is invariably Full Sequential and expected entries decrease uniformly with internalization, consistently with the idea of declining business dynamism as common ownership progresses. At such low volatility levels the probability that the process hits the second and third firm thresholds is extremely small. Expected firm entries and the probability of the first firm's entry then overlap, and as we show further below industry outcomes in terms of consumer surplus and welfare are essentially driven by the first firm's entry. The pattern at intermediate volatility levels (7b) is similar initially, but at λ^* the equilibrium shifts to Duopoly Coordination. The resulting drop in the first entry threshold causes expected entries to jump up, and the simultaneous entry threshold of the last two firms increases enough that expected entries are here also driven by the first firm's entry. Comparing with the other panels, this case also yields higher expected entry than both the lower and higher volatility levels. Finally, at even higher volatility (7c), there is in addition the Clustering equilibrium at intermediate λ values. In this equilibrium also, contest-like dynamics prevail and expected entries increase with internalization, driven this time by the first two firms.

We next study the normative properties of equilibrium. Investment thresholds react to

⁸This probability is equal to $\mathbb{P}\left(\sup_{0 \leq t \leq T} X(t) \geq -\ln\left(\frac{Y}{Y_i}\right) | X(0) = 0\right)$ where $X(t)$ is a Brownian motion $dX(t) = (\mu - \frac{1}{2}\sigma^2)dt + \sigma d\omega(t)$ with $X(0) = 0$. The reflection principle for Brownian motion with drift then yields

$$\mathbb{P}(\tau_i \leq T) = \Phi\left(\frac{\ln(Y/Y_i) + (\mu - \sigma^2/2)T}{\sigma\sqrt{T}}\right) + \left(\frac{Y_i}{Y}\right)^{1-2\mu/\sigma^2} \Phi\left(\frac{\ln(Y/Y_i) - (\mu - \sigma^2/2)T}{\sigma\sqrt{T}}\right),$$

where $\Phi(x)$ is the standard normal cdf.

internalization in opposing ways, so the overall effect on consumers and economic welfare does not have an analytic characterization. The numerical analysis here allows us to nevertheless draw some valuable insights. We suppose that the industry starts from a low enough initial state at which no firm has entered and evaluate the different surpluses.

Denote total industry profit or surplus by IS . Because preemption equalizes rents and rent equalization carries over to private values, industry surplus can be evaluated using only the third firm value expression, which gives

$$IS(Y) = 3V_3^0(Y) = 3 \left(\frac{D_3(\lambda)}{r - \mu} Y_3 - I \right) \left(\frac{Y}{Y_3} \right)^\beta \quad (28)$$

where Y_3 is the relevant third firm threshold, namely $Y_3^*(\lambda)$ if the equilibrium is Full Sequential or Clustering, and $Y_2^S(\lambda)$ with Follower Coordination.

For simplicity we assume that consumer surplus is discounted at the same rate as profits. Consumer surplus is therefore

$$CS(Y) = \frac{S_1}{r - \mu} Y_1 \left(\frac{Y}{Y_1} \right)^\beta + \frac{S_2(\lambda) - S_1}{r - \mu} Y_2 \left(\frac{Y}{Y_2} \right)^\beta + \frac{S_3(\lambda) - S_2(\lambda)}{r - \mu} Y_3 \left(\frac{Y}{Y_3} \right)^\beta \quad (29)$$

where Y_1 , Y_2 , and Y_3 denote the relevant entry thresholds for the first, second and third firms and $S_N(\lambda)$ denotes the equilibrium consumer surplus flow with N firms. With linear demand, the consumer surplus flow is $S_N(\lambda) = \frac{b}{2} Q_N^2(\lambda)$, where $Q_N(\lambda)$ is equilibrium output in Table 1. Finally total welfare is the sum $TS(Y) = IS(Y) + CS(Y)$.

Figure 8 plots the three surplus measures in the same parameter space as Figure 4. Throughout the different panels we can see first of all that volatility (equivalently β) is an important driver of surplus, but also that internalization can exert a pronounced effect through the equilibrium type.

For industry surplus in (8a), profitability is continuous over the Full Sequential and Clustering regions because the last firm's investment threshold is the same in both cases. Over the Follower Coordination region though, industry surplus jumps down as indicated by the discontinuous level curves. The softened investment behavior of the last two firms under coordination therefore leads to inefficiently early entry in the first stage from the firms' perspective. The concavity of the level curves indicates that the optimal level of internalization from the industry's perspective (short of full merger) is positive and lies within the Full Sequential or Clustering regions.

From the perspective of consumers, represented in (8b), the normative properties of the equilibrium types are reversed. Consumers derive a significant benefit from a more intense race to enter first when the last two firms shift to the Follower Coordination equilibrium. This finding is consistent with the rule in Section 3.4 proxying welfare by first firm entry. On the other hand, at lower uncertainty levels (high β) where the equilibrium is invariably Full Sequential, increasing internalization harms consumers.

Lastly, consumer surplus is significantly larger than industry surplus, so total welfare in (8c) displays a similar pattern. In particular, higher common ownership levels have the ability to raise welfare by inducing sufficiently intense competition for the market.

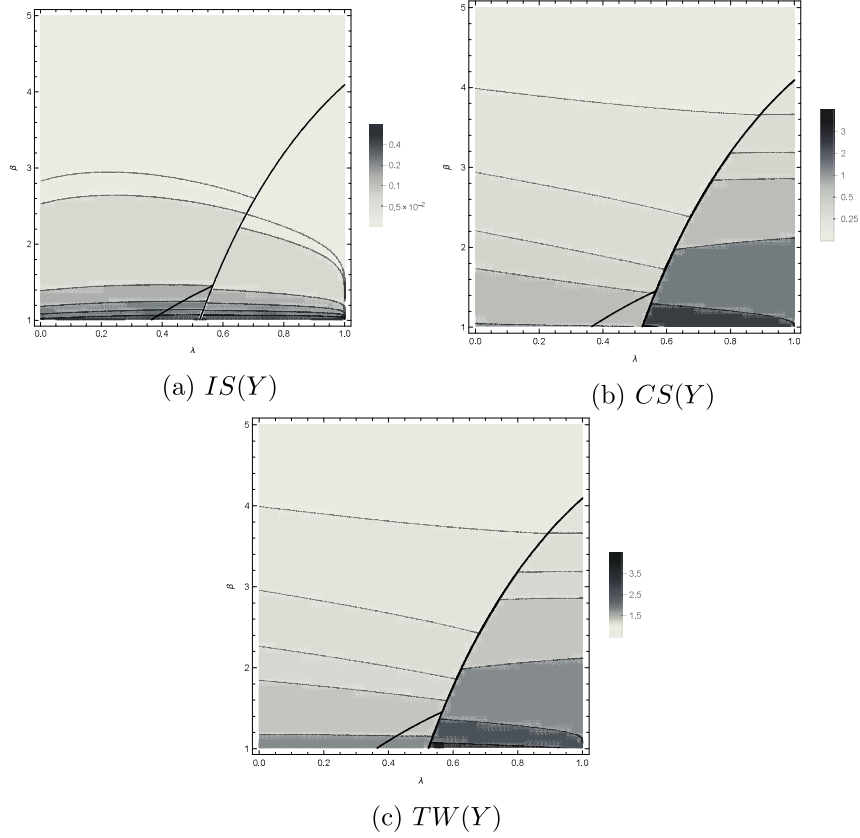


Figure 8: Industry surplus, consumer surplus, and total welfare levels with linear demand, evaluated at $Y = 0.15$. Industry surplus in (8a) jumps down when the equilibrium shifts to Follower Coordination. Conversely, consumer surplus in (8b) jumps up at the Follower Coordination boundary. The sum of the two, total welfare in (8c), also jumps up.

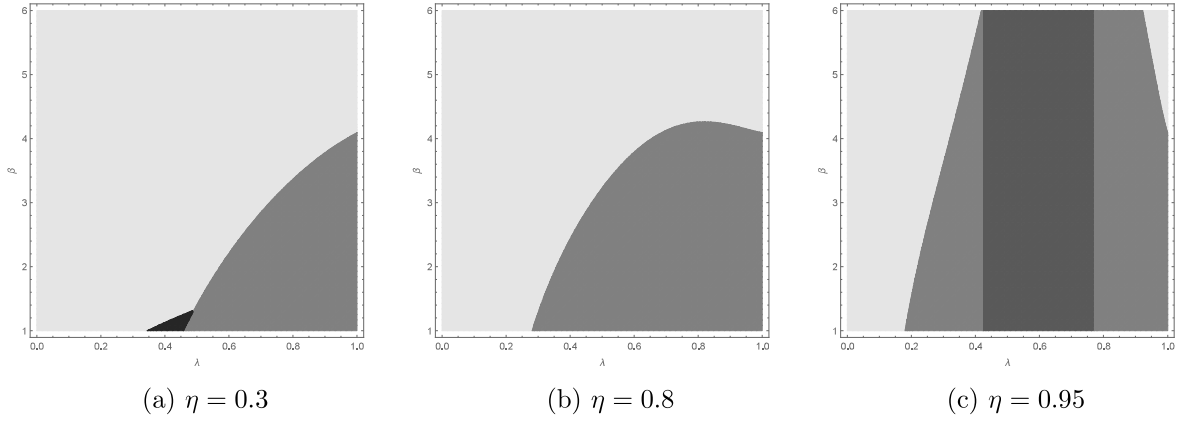


Figure 9: Equilibrium regions under Cournot competition with isoelastic demand, as demand elasticity $\epsilon = 1/\eta$ decreases towards unity. In (9a) the equilibrium pattern is similar to the linear case, with Clustering and Follower Coordination at high internalization and volatility levels and Full Sequential equilibrium otherwise. In (9b) lower demand elasticity expands the Follower Coordination region and Clustering disappears. With near unitary elasticity, in (9c) Follower Coordination expands further and a Monopoly Contest equilibrium emerges at intermediate internalization levels.

4.2 Isoelastic demand

We complement the analysis of linear demand by studying an isoelastic specification. Without loss of generality, we set $A = c = 1$ in the Table 1 payoffs. With this demand and cost, it can be checked that $\Delta_3(\lambda)$ is positive and decreasing for all $\lambda < 1$. Aside from acting as a robustness check, the isoelastic specification allows us to vary the demand curvature $\delta(Q) = -(1 + \eta)$ which regulates the sensitivity of product market competition to internalization. Lower values of η , which reflect higher values of demand elasticity $\epsilon = 1/\eta$, represent a less convex demand, whereas as the parameter η approaches one demand approaches the limiting case of unit elasticity. There are broad similarities between the linear and constant elasticity specifications so we focus our discussion on those aspects where the results they produce diverge.

Figure 9 illustrates equilibrium outcomes in (λ, β) space. Comparing with Figure 4 allows us to assess the overall effect of varying the demand specification. In (9a) we can see that the equilibrium regions are extremely similar to the linear demand regions, with Clustering and Follower Coordination emerging in the bottom right quadrant where internalization and option value are both high. Lowering demand elasticity expands Follower Coordination while Clustering vanishes in (9b). As elasticity approaches one in (9c), the Follower Coordination region continues to expand and a new equilibrium emerges, Monopoly contest, which is not present with linear demand.

The limiting case of unit elasticity provides insight as to why Duopoly Coordination expands over the panels in the figure. As elasticity tends to one, total revenue tends towards a constant, so introducing more firms mainly shifts rents between firms. As (Fudenberg & Tirole, 1985) observe, the value of coordination increases and condition (20) is more likely to hold in such cases.

To get a sense of investment timing in the neighborhood of a Monopoly Contest equilibrium, Figure 10 plots the investment thresholds for a representative volatility level. Unlike with linear demand, the simultaneous investment incentive $\Delta_2^S(\lambda)$ is not monotonically decreasing which

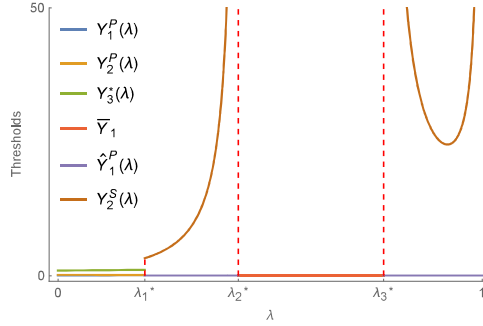


Figure 10: Investment thresholds as function of λ under quantity competition with isoelastic demand and unit marginal costs, $\sigma = 0.25$. Below λ_1^* the equilibrium is Full sequential. The threshold pattern up to λ_2^* is similar to (5b). Between λ_2^* and λ_3^* , is a Monopoly Contest equilibrium with a single threshold, and after λ_3^* we observe a Follower Coordination pattern but the joint threshold of the last two firms is no longer monotonic.

allows it to take negative values where the last two firms abstain from investing by setting infinite thresholds. The mechanism driving simultaneous investment is also related to demand curvature. At low demand elasticity values, profits become highly sensitive to internalization (specifically, the semi-elasticity of profits is inversely related to the curvature δ , see (35)) which can drive the perceived investment incentive in the negative range. When this occurs, the equilibrium reverts to winner-take-all dynamics and we observe the first entry occurring at the most competitive possible trigger, $\bar{Y}_1$.

5 Conclusion

This paper has shown how common ownership can slow down new market entry but can also drastically alter the interaction between firms so contest-like dynamics prevail. By studying entry competition in the context of a three-firm industry, we identify a significantly richer pattern of interactions where all or only some firms may enter and entries may occur sequentially or in different kinds of waves. Internalization intensifies competition between the last two firms by lowering the last firm's investment incentive, but also provides a motive for firms to coordinate their investments at the duopoly stage. In the former case, competition to enter first is softened in line with observed patterns of decreasing business dynamism, but in the latter case competition to be first has a more intense contest-like dynamic. Aside from contributing to the understanding of different forces governing business dynamism, our analysis also speaks to policy debates regarding common ownership by showing that consumer surplus and welfare may actually increase in industries where internalization and volatility produce winner-take-all dynamics. Our analysis relies on simplifying assumptions like symmetry or limits on firm numbers and maintains the focus on entry in a single industry without incorporating exit or resource reallocation which could constitute interesting directions for future work, along with extending the analysis to a general equilibrium framework.

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A Proofs and derivations

A.1 Proof of Proposition 1

In this section we first derive general results for Cournot competition under the assumptions that unit costs, denoted by c , are symmetric and constant, and that inverse demand $P(Q)$ satisfies $P(0) > c$ and is twice differentiable with curvature $\delta(Q) := \frac{P''(Q)Q}{P'(Q)} > -2$.

With N firms, the perceived profit of an individual firm is

$$\pi_i^\lambda(q_i, Q_{-i}) = (P(q_i + Q_{-i}) - c)(q_i + \lambda Q_{-i}). \quad (30)$$

The curvature assumption implies that there is a unique optimal output q_i^* that satisfies the first-order condition $P(q_i^* + Q_{-i}) - c + (q_i^* + \lambda Q_{-i}) P'(q_i^* + Q_{-i}) = 0$. In a symmetric equilibrium, $q_i^* = \frac{Q_N^*}{N}$ and $Q_{-i} = \frac{N-1}{N} Q_N^*$. Substituting these expressions into the first-order condition, equilibrium total output $Q_N^*(\lambda)$ is therefore defined implicitly by

$$\frac{P(Q_N^*) - c}{-Q_N^* P'(Q_N^*)} = \frac{1 + \lambda(N-1)}{N}. \quad (31)$$

Differentiating the left-hand side of (31) with respect to Q_N^* gives

$$\begin{aligned} \left(\frac{P(Q_N^*) - c}{-Q_N^* P'(Q_N^*)} \right)' &= \frac{-Q_N^* P'^2(Q_N^*) + (P(Q_N^*) - c)(P'(Q_N^*) + Q_N^* P''(Q_N^*))}{Q_N^{*2} P'^2(Q_N^*)} \\ &= -\frac{1 + \lambda(N-1)}{N} \frac{1}{Q_N^*} \left(\frac{N + 1 + \lambda(N-1)}{1 + \lambda(N-1)} + \delta(Q_N^*) \right) < 0, \end{aligned} \quad (32)$$

where the expression in parenthesis is positive due to the curvature assumption. Therefore,

$$\frac{dQ_N^*}{d\lambda} = -\frac{(N-1)Q_N^*}{N + 1 + (1 + \lambda(N-1))(\delta(Q_N^*) + 1)} < 0. \quad (33)$$

The profit of an individual firm in equilibrium is

$$D_N(Q_N^*) = -\frac{1 + \lambda(N-1)}{N^2} P'(Q_N^*) Q_N^{*2}. \quad (34)$$

Letting $D_N(\lambda) := D_N(Q_N^*(\lambda))$, it follows that

$$\begin{aligned} D'_N(\lambda) &= -\frac{N-1}{N^2} P'(Q_N^*) Q_N^{*2} - \frac{1 + \lambda(N-1)}{N^2} (2P'(Q_N^*) Q_N^* + P''(Q_N^*) Q_N^{*2}) \frac{dQ_N^*}{d\lambda} \\ &= -\frac{N-1}{N^2} P'(Q_N^*) Q_N^{*2} \frac{(N-1)(1-\lambda)}{N + (1 + \lambda(N-1))(\delta(Q_N^*) + 1)}. \end{aligned} \quad (35)$$

This yields a compact expression for the semi-elasticity of own profit with respect to internalization,

$$\frac{D'_N(\lambda)}{D_N(\lambda)} = \frac{(N-1)^2(1-\lambda)}{(1 + \lambda(N-1))(N + (1 + \lambda(N-1))(\delta(Q_N^*) + 1))}. \quad (36)$$

It is clear from the above that the semi-elasticity is decreasing in equilibrium curvature $\delta(Q_N^*)$, so that as inverse demand becomes more curved all else equal, equilibrium profit becomes less sensitive to internalization changes. The semi-elasticity also provides a convenient sufficient

condition for Assumption C.

Lemma 8. *If $\frac{D'_j(\lambda)}{D_j(\lambda)} \leq 1$ then $\Delta'_{j,i}(\lambda) < 0$.*

Proof. Differentiating gives $\Delta'_{j,i}(\lambda) = (1 + \lambda(j-1)) D'_j(\lambda) - \lambda i D'_i(\lambda) + (j-1) D_j(\lambda) - i D_i(\lambda)$. To obtain an upper bound observe that by Assumption A, $D'_i(\lambda) > 0$ and $(j-1) D_j(\lambda) - i D_i(\lambda) = (j D_j(\lambda) - i D_i(\lambda)) - D_j(\lambda) < 0$. It therefore suffices if $(1 + \lambda(j-1)) D'_j(\lambda) - D_j(\lambda) \leq D'_j(\lambda) - D_j(\lambda) < 0$. $\square$

For two and three firms, we respectively have

$$\frac{D'_2(\lambda)}{D_2(\lambda)} = \frac{1 - \lambda}{(1 + \lambda)(3 + \lambda + (1 + \lambda)\delta(Q_2^*))} \quad (37)$$

which is smaller than one if $\delta(Q) > -2$, and

$$\frac{D'_3(\lambda)}{D_3(\lambda)} = \frac{4(1 - \lambda)}{(1 + 2\lambda)(4 + 2\lambda + (1 + 2\lambda)\delta(Q_3^*))} \quad (38)$$

which is no larger than one if inverse demand is weakly concave.

A.2 Third firm decision problem

The net exercise payoff $\mathbb{E}_Y [\int_0^\infty \Delta_3(\lambda) Y(t)^{-rt}] - I = \frac{\Delta_3(\lambda)}{r-\mu} Y - I$ is linear in the state so the optimal policy is a threshold (Dixit & Pindyck, 1994), which we denote $Y_3^*(\lambda)$. The firm value $F_3(Y)$ satisfies the ODE over the inaction region,

$$\frac{1}{2} \sigma^2 Y^2 F_3''(Y) + \mu Y F_3'(Y) - r F_3(Y) + 2\lambda D_2(\lambda) Y = 0. \quad (39)$$

The lower boundary condition $\lim_{Y \rightarrow 0} F_3(Y) = 0$ suggests a general solution to the ODE of the form $F_3(Y) = \frac{2\lambda D_2(\lambda)}{r-\mu} Y + B Y^\beta$, where $\beta > 1$ is the upper root of the fundamental quadratic $\frac{1}{2} \sigma^2 x(x-1) + \mu x - r = 0$. The solution satisfies the value matching and smooth pasting conditions

$$F_3(Y_3^*) = \frac{(1 + 2\lambda) D_3(\lambda)}{r - \mu} Y_3^* - I \quad (40)$$

and

$$F_3'(Y_3^*) = \frac{(1 + 2\lambda) D_3(\lambda)}{r - \mu}. \quad (41)$$

Substituting the general solution into (40) and (41) and solving for Y_3^* and B gives (12) in the text and $B = \frac{I}{\beta-1} Y_3^{*- \beta}$, from which (13) follows.

A.3 Duopoly subgame

A.3.1 Proof of Proposition 2

Denote the preemption incentive $f_2(Y, \lambda) = V_2^\lambda(Y) - (F_3(Y) - \lambda I)$.

We start with the case $\Delta_3(\lambda) > 0$. Taking the expectation of (14) and subtracting the first

piece of (13) gives

$$\begin{aligned}
f_2(Y, \lambda) &= \frac{(1+\lambda) D_2(\lambda)}{r-\mu} Y - I + \left(\frac{Y}{Y_3^*(\lambda)} \right)^\beta \left(\frac{(1+2\lambda) D_3(\lambda) - (1+\lambda) D_2(\lambda)}{r-\mu} Y_3^*(\lambda) - \lambda I \right) \\
&\quad - \frac{2\lambda D_2(\lambda)}{r-\mu} Y + \lambda I - \left(\frac{Y}{Y_3^*(\lambda)} \right)^\beta \frac{I}{\beta-1} \\
&= (1-\lambda) \left(\frac{D_2(\lambda)}{r-\mu} Y - I - \left(\frac{Y}{Y_3^*(\lambda)} \right)^\beta \left(\frac{D_2(\lambda)}{r-\mu} Y_3^*(\lambda) - I \right) \right). \tag{42}
\end{aligned}$$

Given $\lambda < 1$, the preemption incentive satisfies $f_2(0, \lambda) = -(1-\lambda)I < 0$ and $f_2(Y_3^*(\lambda), \lambda) = 0$. Moreover $\frac{\partial^2 f_2}{\partial Y^2}(Y, \lambda) = -\beta(\beta-1)(1-\lambda) \frac{Y^{\beta-2}}{Y_3^{*\beta}(\lambda)} \left(\frac{D_2(\lambda)}{r-\mu} Y_3^*(\lambda) - I \right) < 0$ so $f_2(Y, \lambda)$ is concave in Y and

$$\begin{aligned}
\frac{\partial f_2}{\partial Y}(Y_3^*(\lambda), \lambda) &= (1-\lambda) \left(\frac{D_2(\lambda)}{r-\mu} - \frac{\beta}{Y_3^*(\lambda)} \left(\frac{D_2(\lambda)}{r-\mu} Y_3^*(\lambda) - I \right) \right) \\
&= -(1-\lambda)(1+2\lambda)(\beta-1) \frac{D_2(\lambda) - D_3(\lambda)}{r-\mu} < 0 \tag{43}
\end{aligned}$$

so the preemption incentive takes positive values immediately left of $Y_3^*(\lambda)$. Viewed as a function of Y , $f_2(Y, \lambda)$ therefore crosses the horizontal axis from below once over $(0, Y_3^*(\lambda))$ defining a unique lower root $Y_2^P(\lambda) = \inf \{Y \in (0, Y_3^*(\lambda)) : f_2(Y, \lambda) > 0\}$ which satisfies (16).

Next, suppose $\Delta_3(\lambda) \leq 0$. Then the follower never enters and the preemption incentive reduces to

$$f_2(Y, \lambda) = (1-\lambda) \left(\frac{D_2(\lambda)}{r-\mu} Y - I \right), \tag{44}$$

implying preemption at the break-even threshold $\bar{Y}_2(\lambda) = \frac{r-\mu}{D_2(\lambda)} I$.

A.3.2 Proof of Proposition 3

Start with the case $\Delta_2^S(\lambda) > 0$ (which implies $\Delta_3(\lambda), \Delta_2(\lambda) > 0$). The value from simultaneous investment (17) follows from a similar argument to Appendix A.2, yielding (19) in the text. Evaluating the difference $F_2^S(Y) - V_2^\lambda(Y)$ gives

$$\begin{aligned}
g(Y, \lambda) &= -\frac{\Delta_2(\lambda)}{r-\mu} Y + I + \frac{(1+\lambda)I}{\beta-1} \left(\frac{Y}{Y_2^S(\lambda)} \right)^\beta \\
&\quad - \left(\frac{(1+2\lambda) D_3(\lambda) - (1+\lambda) D_2(\lambda)}{r-\mu} Y_3^*(\lambda) - \lambda I \right) \left(\frac{Y}{Y_3^*(\lambda)} \right)^\beta. \tag{45}
\end{aligned}$$

We have $g(0, \lambda) = I > 0$ and $\frac{\partial g}{\partial Y}(0, \lambda) = -\frac{\Delta_2(\lambda)}{r-\mu} < 0$, so $g(Y, \lambda)$ is initially positive and decreasing. Next we show that $g(Y, \lambda)$ is convex in Y . Taking the second derivative gives

$$\frac{\partial^2 g}{\partial Y^2}(Y, \lambda) = \frac{\beta}{Y^2} \left(\frac{Y}{Y_3^*(\lambda)} \right)^\beta I \left((1+\lambda)^{1-\beta} \left(\frac{\Delta_2^S(\lambda)}{\Delta_3(\lambda)} \right)^\beta - \beta \frac{(1+2\lambda) D_3(\lambda) - (1+\lambda) D_2(\lambda)}{\Delta_3(\lambda)} + \lambda(\beta-1) \right). \tag{46}$$

The second derivative has the sign of the bracketed term. For $\beta = 1$ this term is equal to $\frac{\Delta_2(\lambda)}{\Delta_3(\lambda)} > 0$. Its derivative with respect to β is

$$(1+\lambda) \ln \left(\frac{\Delta_2^S(\lambda)}{(1+\lambda)\Delta_3(\lambda)} \right) \left(\frac{\Delta_2^S(\lambda)}{(1+\lambda)\Delta_3(\lambda)} \right)^\beta - \frac{(1+2\lambda) D_3(\lambda) - (1+\lambda) D_2(\lambda)}{\Delta_3(\lambda)} + \lambda. \tag{47}$$

We have $\frac{\Delta_2^S(\lambda)}{(1+\lambda)\Delta_3(\lambda)} < 1$ and $\ln(x)x^\beta = (\ln(x)x)x^{\beta-1} > (x-1)x^{\beta-1} > x-1$ for any $x \in (0, 1)$, so

$$(1+\lambda) \ln \left(\frac{\Delta_2^S(\lambda)}{(1+\lambda)\Delta_3(\lambda)} \right) \left(\frac{\Delta_2^S(\lambda)}{(1+\lambda)\Delta_3(\lambda)} \right)^\beta > \frac{\Delta_2^S(\lambda)}{\Delta_3(\lambda)} - (1+\lambda). \quad (48)$$

Then (47) is larger than $\frac{\Delta_2^S(\lambda)}{\Delta_3(\lambda)} - (1+\lambda) - \frac{(1+2\lambda)D_3(\lambda) - (1+\lambda)D_2(\lambda)}{\Delta_3(\lambda)} + \lambda = \frac{\Delta_2(\lambda) - \Delta_3(\lambda)}{\Delta_3(\lambda)} > 0$, so the bracketed term in (46) is positive and therefore that $g(Y, \lambda)$ is convex in Y .

Suppose that $\frac{\partial g}{\partial Y}(Y_3^*(\lambda), \lambda) > 0$, so $g(Y, \lambda)$ has an interior minimum over $(0, Y_3^*(\lambda))$. At such a minimum $\hat{Y}$ we have $\frac{\partial g}{\partial Y}(\hat{Y}, \lambda) = -\frac{\Delta_2(\lambda)}{r-\mu} + \frac{\beta}{\hat{Y}} \left(g(\hat{Y}, \lambda) + \frac{\Delta_2(\lambda)}{r-\mu} \hat{Y} - I \right) = 0$. Solving for $\hat{Y}$ gives

$$\hat{Y} = \Delta_2(\lambda)^{\frac{1}{\beta-1}} \left(\frac{\Delta_2^S(\lambda)^\beta}{\Delta_3(\lambda)^{\beta-1}(1+\lambda)^{\beta-1}} - \beta((1+2\lambda)D_3(\lambda) - (1+\lambda)D_2(\lambda)) + \lambda(\beta-1)\Delta_3(\lambda) \right)^{-\frac{1}{\beta-1}} Y_3^*(\lambda). \quad (49)$$

At the minimum, substituting the first-order condition back into the objective, $g(\hat{Y}, \lambda) = -\frac{\beta-1}{\beta} \frac{\Delta_2(\lambda)}{r-\mu} \hat{Y} + I$. It follows that $g(\hat{Y}, \lambda) > 0$ holds if and only if (20).

Otherwise, suppose that $\frac{\partial g}{\partial Y}(Y_3^*(\lambda), \lambda) \leq 0$. Express $\frac{\partial g}{\partial Y}(Y, \lambda)$ as $\frac{\beta}{Y} \left(g(Y, \lambda) + \frac{\beta-1}{\beta} \frac{\Delta_2(\lambda)}{r-\mu} Y - I \right)$. Then $\frac{\partial g}{\partial Y}(Y_3^*(\lambda), \lambda) \leq 0$ implies $g(Y_3^*(\lambda), \lambda) \leq I - \frac{\beta-1}{\beta} \frac{\Delta_2(\lambda)}{r-\mu} Y_3^*(\lambda) = I \left(1 - \frac{\Delta_2(\lambda)}{\Delta_3(\lambda)} \right) \leq 0$, which implies that there is no simultaneous equilibrium in this case.

Next suppose that $\Delta_2^S(\lambda) \leq 0$ so the last two firms abstain from investing, and that $\Delta_3(\lambda) > 0$. Then

$$g(Y, \lambda) = -\frac{\Delta_2(\lambda)}{r-\mu} Y + I - \left(\frac{(1+2\lambda)D_3(\lambda) - (1+\lambda)D_2(\lambda)}{r-\mu} Y_3^*(\lambda) - \lambda I \right) \left(\frac{Y}{Y_3^*(\lambda)} \right)^\beta. \quad (50)$$

As above $g(Y, \lambda)$ is initially positive and decreasing. Taking the second derivative with respect to Y in this case gives

$$\frac{\partial^2 g}{\partial Y^2}(Y, \lambda) = \frac{\beta}{Y^2} \left(\frac{Y}{Y_3^*(\lambda)} \right)^\beta I \left(\beta \frac{(1+\lambda)D_2(\lambda) - (1+2\lambda)D_3(\lambda)}{\Delta_3(\lambda)} + \lambda(\beta-1) \right). \quad (51)$$

If the second derivative is non-positive, then $g(Y, \lambda)$ takes negative values and there is no simultaneous equilibrium. Otherwise, if $\frac{\partial^2 g}{\partial Y^2}(Y, \lambda) > 0$, we check the value of the global minimum as above. Setting the derivative with respect to Y to zero and solving for Y gives

$$\hat{Y} = \left(\frac{\Delta_2(\lambda)}{\beta((1+\lambda)D_2(\lambda) - (1+2\lambda)D_3(\lambda)) + \lambda(\beta-1)\Delta_3(\lambda)} \right)^{\frac{1}{\beta-1}} Y_3^*(\lambda), \quad (52)$$

and again then $g(\hat{Y}, \lambda) > 0$ holds if and only if (20). If $\Delta_3(\lambda) \leq 0$ finally, then the third firm never enters and $g(Y, \lambda)$ takes negative values, implying there is no simultaneous investment equilibrium in this case.

A.4 Proof of Proposition 4

A.4.1 Existence of $\hat{Y}_1^P(\lambda)$

Suppose the duopoly subgame between the second and third firms admits a simultaneous equilibrium. The preemption incentive is the difference $\hat{f}_1(Y, \lambda) = \hat{V}_1^\lambda(Y) - (F_2^S(Y) - \lambda I)$.

We start with the case $\Delta_2^S(\lambda) > 0$, so the second and third firms enter at a finite threshold.

Then the preemption incentive is

$$\hat{f}_1(Y, \lambda) = (1 - \lambda) \left(\frac{D_1}{r - \mu} Y - I - \left(\frac{Y}{Y_2^S(\lambda)} \right)^\beta \left(\frac{D_1}{r - \mu} Y_2^S(\lambda) - I \right) \right). \quad (53)$$

Given $\lambda < 1$, the preemption incentive satisfies $\hat{f}_1(0, \lambda) = -(1 - \lambda)I < 0$ and $\hat{f}_1(Y_2^S(\lambda), \lambda) = 0$. Moreover $\frac{\partial^2 \hat{f}_1}{\partial^2 Y}(Y, \lambda) = -\beta(\beta - 1)(1 - \lambda) \frac{Y^{\beta-2}}{Y_2^{S\beta}(\lambda)} \left(\frac{D_1}{r - \mu} Y_2^S(\lambda) - I \right)$ and

$$\begin{aligned} \frac{\partial \hat{f}_1}{\partial Y}(Y_2^S(\lambda), \lambda) &= (1 - \lambda) \left(\frac{D_1}{r - \mu} - \frac{\beta}{Y_2^S(\lambda)} \left(\frac{D_1}{r - \mu} Y_2^S(\lambda) - I \right) \right) \\ &= -(1 - \lambda)(\beta - 1) \frac{D_1}{r - \mu} \left(1 - \frac{Y_1^*}{Y_2^S(\lambda)} \right), \end{aligned} \quad (54)$$

where $Y_1^* = \frac{\beta}{\beta-1} \frac{r-\mu}{D_1} I$ is the monopoly investment threshold. Because $Y_2^S(\lambda) > Y_1^*$, we have $\frac{\partial^2 \hat{f}_1}{\partial^2 Y}(Y, \lambda) < 0$ and $\frac{\partial \hat{f}_1}{\partial Y}(Y_2^S(\lambda), \lambda) < 0$ so $\hat{f}(Y, \lambda)$ is concave in Y over $(0, Y_2^S(\lambda))$ and decreasing at $Y_2^S(\lambda)$, implying a unique lower root $\hat{Y}_1^P(\lambda) = \inf \left\{ Y \in (0, Y_2^S(\lambda)) : \hat{f}_1(Y, \lambda) > 0 \right\}$ which satisfies (25).

Next, if $\Delta_2^S(\lambda) \leq 0$, the last two firms abstain from ever investing and the preemption incentive reduces to

$$\hat{f}_1(Y, \lambda) = (1 - \lambda) \left(\frac{D_1}{r - \mu} Y - I \right), \quad (55)$$

which is negative for $Y \geq \bar{Y}_1 = \frac{(r-\mu)I}{D_1}$.

A.4.2 Existence of $Y_1^P(\lambda)$

Suppose the duopoly subgame between the second and third firms is preemptive, so the second firm enters at $Y_2^P(\lambda)$. The preemption incentive is the difference $f_1(Y, \lambda) = V_1^\lambda(Y) - (V_2^\lambda(Y) - \lambda I)$.

We start with the case $\Delta_3(\lambda) > 0$, so the third firm enters at a finite threshold. Taking the expectation of (22) and subtracting then the expectation of (14) gives

$$\begin{aligned} &\frac{D_1}{r - \mu} Y - I + \left(\frac{Y}{Y_2^P(\lambda)} \right)^\beta \left(\frac{(1 + \lambda) D_2(\lambda) - D_1}{r - \mu} Y_2^P(\lambda) - \lambda I \right) \\ &+ \left(\frac{Y}{Y_3^*(\lambda)} \right)^\beta \left(\frac{(1 + 2\lambda) D_3(\lambda) - (1 + \lambda) D_2(\lambda)}{r - \mu} Y_3^*(\lambda) - \lambda I \right) \\ &- \frac{\lambda D_1}{r - \mu} Y + \lambda I - \left(\frac{Y}{Y_2^P(\lambda)} \right)^\beta \left(\frac{(1 + \lambda) D_2(\lambda) - \lambda D_1}{r - \mu} Y_2^P(\lambda) - I \right) \\ &- \left(\frac{Y}{Y_3^*(\lambda)} \right)^\beta \left(\frac{(1 + 2\lambda) D_3(\lambda) - (1 + \lambda) D_2(\lambda)}{r - \mu} Y_3^*(\lambda) - \lambda I \right). \end{aligned} \quad (56)$$

Simplifying,

$$f_1(Y, \lambda) = (1 - \lambda) \left(\frac{D_1}{r - \mu} Y - I - \left(\frac{Y}{Y_2^P(\lambda)} \right)^\beta \left(\frac{D_1}{r - \mu} Y_2^P(\lambda) - I \right) \right). \quad (57)$$

Given $\lambda < 1$, the preemption incentive satisfies $f_1(0, \lambda) = -(1 - \lambda)I < 0$ and $f_1(Y_2^P(\lambda), \lambda) = 0$.

Moreover $\frac{\partial^2 f_1}{\partial Y^2}(Y, \lambda) = -\beta(\beta - 1)(1 - \lambda) \frac{Y^{\beta-2}}{Y_2^{P\beta}(\lambda)} \left(\frac{D_1}{r-\mu} Y_2^P(\lambda) - I \right) < 0$ and

$$\begin{aligned} \frac{\partial f_1}{\partial Y}(Y_2^P(\lambda), \lambda) &= (1 - \lambda) \left(\frac{D_1}{r - \mu} - \frac{\beta}{Y_2^P(\lambda)} \left(\frac{D_1}{r - \mu} Y_2^P(\lambda) - I \right) \right) \\ &= -(1 - \lambda)(\beta - 1) \frac{D_1}{r - \mu} \left(1 - \frac{Y_1^*}{Y_2^P(\lambda)} \right). \end{aligned} \quad (58)$$

The preemption incentive is therefore concave and takes positive values below $Y_2^P(\lambda)$ if and only if $Y_2^P(\lambda) > Y_1^*$. Suppose for now that this inequality holds (we characterize further below when this is the case). Then there is a unique lower root $Y_1^P(\lambda) = \inf \{Y \in (0, Y_2^P(\lambda)) : f_1(Y, \lambda) > 0\}$, which satisfies (26).

Next if $\Delta_3(\lambda) \leq 0$, it is never profitable for the third firm to enter. Then the duopoly equilibrium involves investment by the second firm only, at the breakeven trigger $\bar{Y}_2(\lambda) = \frac{r-\mu}{D_2(\lambda)} I$. The preemption incentive at the initial stage is still given by (57) and the reasoning is the same.

A.4.3 Clustering of first two firms

As discussed in the text, preemption in the second stage results in a sequential equilibrium with the first two firms investing at their respective triggers $Y_1^P(\lambda) < Y_2^P(\lambda)$ provided that the second firm's preemption trigger lies above the first firm's monopoly threshold, Y_1^* (otherwise there is no incentive to invest ahead of the second firm). If $\Delta_3(\lambda) > 0$, we check this by verifying that the duopoly preemption incentive (42) is negative at Y_1^* , i.e.

$$\frac{D_2(\lambda)}{r - \mu} Y_1^* - I < \left(\frac{Y_1^*}{Y_3^*(\lambda)} \right)^\beta \left(\frac{D_2(\lambda)}{r - \mu} Y_3^*(\lambda) - I \right), \quad (59)$$

and if $\Delta_3(\lambda) \leq 0$ so the third firm never invests the condition reduces to $\frac{D_2(\lambda)}{r-\mu} Y_1^* - I < 0$. Rearranging these inequalities gives (24) in the text.

A.5 Proof of Proposition 5

For part (i), $Y_1^P(\lambda) < Y_1^*$ is immediate. To show that $Y_1^P(\lambda) > Y_1^{(2)P}(\lambda)$, we compare the preemption condition defining $Y_1^P(\lambda)$ with the condition defining the duopoly investment trigger (see footnote 7). The two conditions differ only with respect to the follower firm threshold, which takes the value $Y_2^P(\lambda)$ in the three-firm industry and $Y_2^*(\lambda)$ in the two-firm industry. Because $Y_2^P(\lambda) < Y_2^*(\lambda)$, the result follows (and similarly in the limiting case of $\bar{Y}_2$ if $\Delta_3(\lambda) \leq 0$).

For part (ii), we have $\bar{Y}_1 \leq \hat{Y}_1^P(\lambda)$ and $\hat{Y}_1^P(\lambda) < Y_1^*$ is immediate. For the other inequality, observe that $\Delta_2^S(\lambda) < \Delta_3(\lambda) < (1 + \lambda)\Delta_2(\lambda)$, so the follower firm thresholds in the respective preemption conditions satisfy $Y_2^S(\lambda) > Y_2^*(\lambda)$. The inequality again follows by comparing the preemption condition (25) with the relevant two-firm industry preemption condition in the footnote.

A.6 Proof of Proposition 6

For the second and third firm to invest simultaneously we require (24) to hold. Provided that $\Delta_3(\lambda) > 0$, rewriting this as $D_2(\lambda) - D_2^*(\lambda) \geq 0$ where $D_2^*(\lambda) = \frac{\beta-1}{\beta} \frac{D_1^\beta - \Delta_3(\lambda)^\beta}{D_1^{\beta-1} - \Delta_3(\lambda)^{\beta-1}}$ and taking the derivative with respect to λ we obtain:

$$\underbrace{D_2'(\lambda)}_{(+)} - \underbrace{\beta \frac{\Delta_3(\lambda)^\beta}{D_1^\beta - \Delta_3(\lambda)^\beta} \frac{D_2^*(\lambda)}{\Delta_3(\lambda)^2}}_{(+)} \Delta_3'(\lambda) (D_2^*(\lambda) - \Delta_3(\lambda)).$$

The difference $D_2^*(\lambda) - \Delta_3(\lambda)$ can be written as $\beta - 1 - \beta \frac{\Delta_3(\lambda)}{D_1} + \left(\frac{\Delta_3(\lambda)}{D_1}\right)^\beta$. The expression $\beta - 1 - \beta x + x^\beta$ is always positive for $x \in (0, 1)$ so, provided that $\Delta_3'(\lambda) < 0$, the derivative is positive.

A.7 Proof of Proposition 7

We first detail the key step, $\frac{dY_2^P}{d\lambda} < 0$, then establish the other signs as per the discussion in the text.

A.7.1 Sensitivity of $Y_2^P(\lambda)$ to λ

Applying the implicit function theorem to (16) yields $\frac{dY_2^P}{d\lambda} = -\frac{\frac{\partial f_2}{\partial \lambda}(Y_2^P, \lambda)}{\frac{\partial f_2}{\partial Y}(Y_2^P, \lambda)}$. Because Y_2^P is the lower root, $\frac{\partial f_2}{\partial Y}(Y_2^P, \lambda) > 0$. The sign of the derivative is therefore the opposite of the numerator. We have

$$\begin{aligned} \frac{\partial f_2}{\partial \lambda}(Y, \lambda) &= -\frac{f_2(Y, \lambda)}{1 - \lambda} + (1 - \lambda) \left(\left(\frac{Y}{r - \mu} - \frac{Y_3^*(\lambda)}{r - \mu} \left(\frac{Y}{Y_3^*(\lambda)} \right)^\beta \right) D_2'(\lambda) \right. \\ &\quad \left. - \left(\frac{Y}{Y_3^*(\lambda)} \right)^\beta \left((1 - \beta) \frac{D_2(\lambda)}{r - \mu} + \beta \frac{I}{Y_3^*(\lambda)} \right) Y_3^{*'}(\lambda) \right). \end{aligned} \quad (60)$$

Because $f_2(Y_2^P, \lambda) = 0$, $\frac{\partial f_2}{\partial \lambda}(Y_2^P, \lambda)$ has the sign of the second summand. We have $D_2'(\lambda) > 0$, and $Y_3^{*'}(\lambda) > 0$ due to $\Delta_3'(\lambda) < 0$. Evaluating the sign of the other terms at Y_2^P gives $\frac{Y_2^P(\lambda)}{r - \mu} - \frac{Y_3^*(\lambda)}{r - \mu} \left(\frac{Y_2^P(\lambda)}{Y_3^*(\lambda)} \right)^\beta = \frac{Y_2^P(\lambda)}{r - \mu} \left(1 - \left(\frac{Y_2^P(\lambda)}{Y_3^*(\lambda)} \right)^{\beta - 1} \right) > 0$ and $(1 - \beta) \frac{D_2(\lambda)}{r - \mu} + \beta \frac{I}{Y_3^*(\lambda)} = \frac{\beta - 1}{r - \mu} (\Delta_3(\lambda) - D_2(\lambda)) < 0$, from which $\frac{\partial f_2}{\partial \lambda}(Y_2^P, \lambda) > 0$ and the result follows.

A.7.2 Other threshold statics

For the Sequential duopoly equilibrium, we have $\bar{Y}_2(\lambda) = \frac{r - \mu}{D_2(\lambda)} I$ so the sign of $\frac{d\bar{Y}_2}{d\lambda}$ follows from $D_2'(\lambda) > 0$.

For the Clustering equilibrium, λ affects Y_1^P only via Y_2^P so the sign of $\frac{dY_1^P}{d\lambda}$ follows from similar reasoning to $\frac{dY_2^P}{d\lambda}$.

For the Follower Coordination equilibrium, the opposing signs of $\frac{dY_2^S}{d\lambda}$ and $\hat{Y}_1^P$ follow from the condition (25) via $Y_2^S(\lambda)$. The left hand side is a concave function of Y and $Y_2^S(\lambda)$ lies in the decreasing range, so increasing Y_2^S lowers $\hat{Y}_1^P$ and vice-versa.

For the full accordion effect finally, the third firm effect $\frac{dY_3^*}{d\lambda} > 0$ follows from decreasing investment incentive (Assumption C). The parameter λ affects the preemption threshold Y_2^P directly via $D_2(\lambda)$ and indirectly via $Y_3^*(\lambda)$, but because $D_2'(\lambda) > 0$ and $\frac{dY_3^*}{d\lambda} > 0$ the two effects operate in the same direction, increasing the incentive to preempt and therefore $\frac{dY_2^P}{d\lambda} < 0$.