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Overlapping Ownership, Sequential Moves, and Welfare

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Abstract

We introduce overlapping ownership into a sequential-move duopoly and find conditions where internalization has positive welfare effects in contrast with other standard oligopoly models. Internalization of rival profits softens follower output and entry decisions, inducing more aggressive leader behavior. In Stackelberg equilibrium follower internalization raises total output and welfare provided the follower is active. The effect is stronger if demand is more convex which makes the follower less responsive. In an entry-deterrence setting, higher symmetric internalization that shifts equilibrium from accommodation to deterrence raises welfare relative to a benchmark with independent firms. Product differentiation attenuates the first effect but is necessary for the second effect if firms compete in prices.

JEL Classification: D25, G32, L13.

Keywords: entry deterrence, overlapping ownership, Stackelberg.

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1 Introduction

The rise of large institutional investors has led to widespread horizontal ownership concentrations of publicly listed firms ([Backus et al. 2021](#)). In parallel many industries exhibit significant patterns of cross holdings ([Huse et al. 2024](#)). These concurrent phenomena, known as overlapping ownership, have sparked concern that firms may internalize rival profits and deviate from profit maximization, with adverse consequences for economic welfare ([Posner et al. 2016](#)). Standard oligopoly theory broadly supports such concerns, as under quantity competition, price competition, or repeated oligopoly, equilibrium outcomes are less competitive if firms internalize rival payoffs ([Reynolds and Snapp 1986](#), [Gilo et al. 2006](#), [Shelegia and Spiegel 2012](#)). The link between ownership structure and market outcomes is more involved however if firms compete asymmetrically. Because some firms behave less competitively, others can profit by acting more aggressively so it is not a foregone conclusion that overlapping ownership should always exert an anticompetitive effect on oligopoly outcomes.

To investigate the effect of overlapping ownership on markets where oligopolists compete asymmetrically, we introduce internalization into a sequential-move duopoly framework that is standard in all other respects. In addition to their asymmetric roles, we allow firms to have asymmetric ownership which lets us isolate effects arising from leader and follower internalization. We allow for elements of both common ownership and cross-ownership in our model, in line with observed industry patterns ([Huse et al. 2024](#)). To facilitate interpretation of our results we introduce a stylized ownership structure where common ownership induces a pattern of symmetric internalization and firms also may hold minority stakes that induce asymmetric internalization. While simultaneous-move oligopoly models are often favored in industrial organization – duopolists generally prefer

simultaneous over sequential moves if given the chance ([Amir and Grilo 1999](#))— industry conditions may preclude choosing the timing of interactions and compel firms to move sequentially ([Cumbul 2022](#)). Indeed in several industries where overlapping ownership has been actively studied like airlines or pharmaceuticals, firm interactions in specific sub-markets are often more consistent with sequential moves than simultaneous. Examples include competition on city pairs between an established carrier and a low-cost airline, or between a branded pharmaceutical and a generic entrant. Such interactions are the focus of our analysis.

Our model highlights how leaders balance opposing forces, seeking to capitalize on softer follower behavior on the one hand while being restrained by their own internalization of followers on the other. With this balance in mind we study how internalization affects equilibrium outcomes in the two canonical sequential-move frameworks: Stackelberg competition and entry deterrence. Our analysis reveals two mechanisms through which overlapping ownership can exert a procompetitive effect on markets, in contrast with the conventional wisdom drawn from simultaneous move oligopoly models.

The first mechanism relates to Stackelberg equilibrium. We show that, provided that the follower remains active, softer follower behavior due to overlapping ownership prompts the leader to be sufficiently more aggressive that total output and welfare increase. The aggressiveness of the leader’s response hinges on the follower’s response elasticity, itself driven by demand curvature in our model, and we show that the effect is stronger for more convex demands. With sufficient follower internalization, the effect dissipates and the follower is eventually shut out of the market even though the leader does not practice deterrence in the conventional sense. These effects arise if there are higher follower cross-ownership stakes, but also if common ownership levels rise provided the leader does not have too large a cross-ownership stake. Our findings imply, for example, that markets

where a leader has a cross-ownership stake are more efficient if the follower holds a cross-ownership stake as well.

The second mechanism we identify relates to the leader’s strategy in an entry deterrence framework. We show that greater follower internalization increases the likelihood of deterrence, another form of aggressive leader behavior which shifts market outcomes qualitatively. Relative to a benchmark setup where firms do not internalize and the equilibrium involves accommodation, we show there are positive internalization levels that drive the leader to opt for strategic deterrence and which can result in greater welfare. In the case of linear demand for example, a moderate rise in common ownership levels is procompetitive relative to the independent firms benchmark, because output jumps if the leader deters and also because the industry economizes on entry costs.

We subsequently extend our analysis to differentiated goods. With sequential quantity choices, the procompetitive effects we identify are strongest when products are close substitutes. If firms compete in price and products are differentiated, then the second entry deterrence effect arises whereas it is absent with homogeneous goods. Finally we also show a similar pattern arises if more than two firms are introduced into the model.

1.1 Related literature

Early overlapping ownership work primarily highlighted anticompetitive effects on product market competition. As discussed above, more recent research has also identified positive implications. In the context of R&D decisions for example, internalization can lead to increased investment in the presence of spillovers ([López and Vives 2019](#)). These positive effects occur because softened product market competition raises ex-ante investment incentives, and appear to be borne out by empirical evidence of increased innovative investment under overlapping ownership ([Antón et al. 2024](#)).

Our paper is closely related to a stream of research on entry decisions under overlapping ownership. Earlier work established an inverted-U relationship between overlapping ownership and welfare in a symmetric entry model ([Sato and Matsumura 2020](#)), though more recent research using a more general framework has found higher overlapping ownership harms welfare unless there are significant economies of scale ([Vives and Vivasinos 2025](#)). In contrast with our model, these studies consider firms with symmetric roles so they do not discriminate between ownership links of incumbents and entrants. Another prior study of entry demonstrates that an incumbent firm can use cross-ownership to deter a rival firm’s entry ([Li et al. 2015](#)). The mechanism in that paper hinges on an atypical ownership stake strategy however, with firms unable to make other strategic commitments and ultimately competing à la Cournot. Some further research into how overlapping ownership bears on entry decisions has studied vertical market structures. Downstream incumbents may use backward ownership to deter rival entry for example ([Hunold 2020](#)), though horizontal overlapping ownership in successive oligopoly appears to generate either excessive or insufficient entry ([Basak 2025](#)).

Our findings highlight positive effects of internalization in markets where firms have strategic asymmetries, which is especially relevant in situations of passive partial ownership. Similarly to our finding that internalization can raise welfare by driving the leader to pursue a more aggressive strategy, some prior research in this literature has shown that higher levels of asymmetric internalization can cause welfare-increasing shifts in equilibrium, for example by prompting leading firms to transfer or license their technology ([Leonardos et al. 2021](#)), or by destabilizing collusion ([Pal and Petrakis 2025a](#)).

Empirical studies provide mixed evidence regarding the relationship between overlapping ownership and entry. While some work finds overlapping ownership reduces entry in some segments of the pharmaceutical sector ([Newham et al. 2025](#)), other work shows that

in some market segments, product introductions in general consumer goods markets increase with overlapping ownership (Aslan 2023). By incorporating both softened follower entry incentives and increased leader investment in a unified framework, our analysis goes part of the way to reconcile these contrasting findings.

Another stream of literature studying overlapping ownership and sequential moves has addressed whether or not firms prefer to retain or to divest from cross-ownership stakes. A key finding is that Cournot firms generally have unilateral incentives to divest (Stenbacka and Van Moer 2021), but this result does not extend to sequential-move games generally (Pal and Petrakis 2025b). As discussed above though we see our model as applying most closely to multimarket firms in industries like automobiles or pharmaceuticals. For such firms, ownership stakes are more likely to be driven by broader corporate strategies rather than the desire to shift equilibrium outcomes in a single market, so we take ownership choices to be exogenous with respect to our analysis here.

2 Model setup

We consider an industry described by the standard Stackelberg-Spence-Dixit framework. The demand for a homogeneous product is represented by an inverse demand function

$$P(Q) = A - BQ^{1+\delta} \tag{1}$$

where Q denotes total output, $A, B > 0$ are constants, and $\delta > -1$. This specification parametrizes demand curvature by δ and nests linear, concave and convex demands.

The market is served by up to two firms. These firms have identical technologies and produce with constant unit cost $C \in [0, A)$. To streamline subsequent expressions, we define $\tilde{A} := \frac{A-C}{B}$.

Firms choose their outputs sequentially, and we denote the output of the leader and follower by q_L and q_F respectively. Further below we also introduce a positive entry cost K , assumed not to be prohibitively high so entry is viable, and incorporate an entry decision for the second firm. We solve both versions of the game by backward induction.

The industry has an overlapping ownership structure that involves elements of both common and cross-ownership. The ownership structure is set exogenously before output and entry decisions are made. To fix ideas, we posit an ownership structure along the following lines.¹

Each firm's equity is divided equally among N private owners, apart from any cross-ownership stake held by the other firm. The total owner population at each firm consists of $0 \leq M \leq N$ diversified owners, who hold stakes in both of the firms, and $N - M$ undiversified owners, who hold stakes in a single firm only. In addition to this common ownership layer, we allow each firm to hold a passive, or non-controlling stake s_F or s_L in the other, with $s_F s_L < 1$.

To derive the firm objectives implied by the ownership structure, first denote the standalone follower and leader profits by π_F and π_L . The total values of the follower and the leader, Π_F and Π_L , are sums of each firm's standalone profit and stake in the other's profit, implying that $\Pi_F = \frac{\pi_F + s_F \pi_L}{1 - s_F s_L}$ and $\Pi_L = \frac{s_L \pi_F + \pi_L}{1 - s_F s_L}$ which are standard formulations in cross-ownership models (e.g. [Shelegia and Spiegel 2024](#)).²

Because cross-ownership stakes are assumed to be passive, firms respond to their direct shareholders only. In line with the common ownership literature, we suppose that these owners exert control on the firm proportionally to their ownership shares in the firm, though because the ownership shares are symmetric the result is the same as with

¹Readers may refer to the online appendix of [López and Vives \(2019\)](#) or [Brito et al. \(2023\)](#) for more comprehensive analysis of how financial interests and control rights induce rival firm internalization.

²Specifically, $\Pi_F = \pi_F + s_F \Pi_L$ and $\Pi_L = \pi_L + s_L \Pi_F$, and solving jointly for Π_F and Π_L yields the expressions in the text.

outright wealth maximization.

We first derive the objective of the follower firm. Its undiversified shareholders each have wealth $W^U = \frac{1}{N}(1 - s_L)\Pi_F$, and its diversified shareholders each have wealth $W^D = \frac{1}{N}(1 - s_L)\Pi_F + \frac{1}{N}(1 - s_F)\Pi_L$. The follower firm's objective under proportional control is therefore $\max \left[\sum_{i=1}^N \gamma_{iF} W_i \right]$, where $\gamma_{iF} = \frac{1}{N}$ is the control share of each direct shareholder and $W_i \in \{W^D, W^U\}$ is that shareholder's wealth. Setting $\lambda = \frac{M}{N}$, substituting wealth expressions, and summing gives $\max \left[\frac{1}{N} ((1 - s_L)\Pi_F + \lambda(1 - s_F)\Pi_L) \right]$. Finally substituting the value expressions Π_F and Π_L and normalizing by $\frac{1-s_L+\lambda s_L(1-s_F)}{N(1-s_F s_L)}$ yields the follower's objective,

$$\max [\pi_F + \lambda_F \pi_L] \quad (2)$$

where $\lambda_F := \frac{s_F + \phi_F \lambda}{1 + s_L \phi_F \lambda}$ is the follower's effective degree of internalization of the leader, with $\phi_F = \frac{1-s_F}{1-s_L}$ measuring the relative importance of follower outside ownership.

If cross-ownership shares are zero, i.e. $s_F = s_L = 0$, then the follower internalization parameter λ_F reduces to the standard symmetric common ownership internalization λ , while if $M = 0$ so there are no common owners, then $\lambda_F = s_F$ which is the standard parameter under pure cross-ownership obtained by normalizing the expressions Π_F and Π_L . The effect of common ownership is weighted by the relative importance of outside ownership for the follower firm ϕ_F . We have λ_F increasing in s_F , decreasing in s_L , and increasing in λ . Then, letting $\lambda \rightarrow 1$ establishes that $\lambda_F \leq 1$.

Similar reasoning yields

$$\max [\pi_L + \lambda_L \pi_F] \quad (3)$$

for the leader, where $\lambda_L := \frac{s_L + \phi_L \lambda}{1 + s_F \phi_L \lambda}$ is the leader's effective degree of internalization of the follower, with $\phi_L = \frac{1-s_L}{1-s_F}$ measuring the relative importance of the leader's outside ownership. As above we have λ_L decreasing in s_F , increasing in s_L and λ , and bounded

above by one.

3 Equilibrium analysis

We next characterize the equilibrium effects of overlapping ownership in the two versions of the model, Stackelberg competition and entry deterrence, and show how in both frameworks internalization can have a procompetitive effect on market outcomes.

3.1 Stackelberg equilibrium and first mechanism

We first solve the follower's output choice problem. The follower chooses q_F to maximize its objective (2) taking the leader's output as given, i.e.

$$\max_{q_F \geq 0} [\pi_F(q_F, q_L) + \lambda_F \pi_L(q_F, q_L)]. \quad (4)$$

If $\lambda_F = 1$, the follower maximizes joint profit by setting output $q_F^*(q_L, 1) = \max\{q^M - q_L, 0\}$, where $q^M = \left(\frac{\tilde{A}}{2+\delta}\right)^{\frac{1}{1+\delta}}$ denotes the monopoly output. The leader's optimal output is then simply q^M if it internalizes partially, or any output in $[0, q^M]$ if $\lambda_L = 1$. Hereafter, we focus on the more interesting case where the follower internalizes only partially, $\lambda_F < 1$. The follower's objective is then strictly concave in q_F , and therefore admits a unique maximum whose form depends on the leader's output. Let $\bar{q}(\lambda_F) = \left(\frac{\tilde{A}}{1+\lambda_F(1+\delta)}\right)^{\frac{1}{1+\delta}} > q^M$ denote the critical leader output level below which the follower produces positive output. For leader outputs below this critical level, the follower's optimal output $q_F^*(q_L, \lambda_F)$ is defined implicitly by the first-order condition

$$\tilde{A} - (q_F^* + q_L)^\delta ((2 + \delta)q_F^* + (1 + \lambda_F(1 + \delta))q_L) = 0, \quad (5)$$

in which case we say that the follower is active. Above the critical leader output, the follower's optimal output is zero and the follower is said to be inactive. It is thus possible with internalization for the leader to shut the follower out by producing high enough output and still earn positive profit, even though firms have symmetric costs and there is no entry cost for the follower. At leader outputs where the follower is active, the internalization term in (5) has a negative coefficient implying that higher internalization softens the follower's response, resulting in lower output choice.

We next turn to the leader's output choice problem. The leader chooses q_L to maximize its objective (3) taking the follower's reaction $q_F^*(q_L, \lambda_F)$ as given, i.e

$$\max_{q_L \geq 0} [\pi_L(q_F^*(q_L, \lambda_F), q_L) + \lambda_L \pi_F(q_F^*(q_L, \lambda_F), q_L)]. \quad (6)$$

The leader's objective function is defined piecewise due to the follower's piecewise reaction, so two kinds of optima arise depending on whether or not the follower is active. In equilibria with an active follower, the leader's optimal output satisfies the first-order condition

$$\begin{aligned} \tilde{A} - (q_F^*(q_L^*, \lambda_F) + q_L^*)^\delta ((1 + \lambda_L(1 + \delta))q_F^*(q_L^*, \lambda_F) + (2 + \delta)q_L^*) \\ - (1 - \lambda_F \lambda_L)(1 + \delta)(q_F^*(q_L^*, \lambda_F) + q_L^*)^\delta q_L^* \frac{\partial q_F^*}{\partial q_L}(q_L^*, \lambda_F) = 0, \end{aligned} \quad (7)$$

where the second line uses the follower's first-order condition (5) to simplify the follower reaction slope terms in the leader's marginal profit. As with the follower's first-order condition, the leader's internalization terms here have negative coefficients implying that the leader lowers its output, all else equal, if its internalization level increases. Follower internalization only enters the leader's first-order indirectly, by both lowering the level of

the follower's reaction and altering its slope. In equilibria where the follower is inactive, the leader's marginal profit is identical to the marginal profit of a monopoly firm and its optimal output is therefore a corner solution at the critical cutoff $\bar{q}(\lambda_F)$.

Together (5) and (7) define a unique total output level in equilibria where the follower is active, which admits a closed form expression. In a Stackelberg equilibrium, we find that total output is either

$$Q^S(\lambda_F, \lambda_L) = \begin{cases} \left(\frac{\delta \tilde{A}}{-1 + \frac{1+\delta}{2}(-\Gamma + \sqrt{(2+\Gamma)^2 + 4\delta\Gamma})} \right)^{\frac{1}{1+\delta}} & \lambda_F < \frac{1}{2\delta} \left(-(2+\Gamma) + \sqrt{(2+\Gamma)^2 + 4\delta\Gamma} \right) \\ \bar{q}(\lambda_F) & \lambda_F \geq \frac{1}{2\delta} \left(-(2+\Gamma) + \sqrt{(2+\Gamma)^2 + 4\delta\Gamma} \right) \end{cases}, \quad (8)$$

if $\delta \neq 0$, or

$$Q^S(\lambda_F, \lambda_L) = \begin{cases} \frac{2+\Gamma}{2(1+\Gamma)} \tilde{A} & \lambda_L \lambda_F^2 - (3 - \lambda_L) \lambda_F + 1 > 0 \\ \frac{1}{1+\lambda_F} \tilde{A} & \lambda_L \lambda_F^2 - (3 - \lambda_L) \lambda_F + 1 \leq 0 \end{cases}, \quad (9)$$

if $\delta = 0$,

where $\Gamma(\lambda_F, \lambda_L) = \frac{1-\lambda_F\lambda_L}{1-\lambda_L}$ (see Section A.1).

Products are homogeneous in our model, so consumer surplus is directly related to total output. Because firms have identical and constant unit cost, so is total welfare.³ The total output expressions (8) and (9) allow us to determine directly the effect of each firm's internalization level on equilibrium output, consumer surplus, and welfare. The following proposition states the result.

Proposition 1. *In a Stackelberg equilibrium with overlapping ownership:*

(i) *total output, consumer surplus, and welfare are (strictly) increasing in follower internalization if the follower is active (and $\lambda_L \in (0, 1)$) and strictly decreasing in follower*

³The specific form of total welfare at the Stackelberg equilibrium is $W(\lambda_F, \lambda_L) = \int_0^{Q^S(\lambda_F, \lambda_L)} (A - BQ^{1+\delta} - C) dQ = B \left(\tilde{A} - \frac{Q^{S^{1+\delta}}(\lambda_F, \lambda_L)}{2+\delta} \right) Q^S(\lambda_F, \lambda_L)$.

internalization if the follower is inactive;

(ii) total output, consumer surplus, and welfare are (strictly) decreasing in leader internalization (if the follower is active).

Proof. We start with the $\delta \neq 0$ case. To determine the effect of leader and follower internalization if the follower is active, differentiate the first piece of (8) with respect to Γ to get $\frac{1}{2\delta A} \left(1 - \frac{2+\Gamma+2\delta}{\sqrt{(2+\Gamma)^2+4\delta\Gamma}} \right) Q^{S2+\delta} d\Gamma$. We have $\frac{2+\Gamma+2\delta}{\sqrt{(2+\Gamma)^2+4\delta\Gamma}} < 1$ if and only if $4\delta(2+\delta) < 0$, so the product of the first two terms is negative. The sign of the entire expression is therefore the opposite of $d\Gamma$. For λ_F , $\frac{\partial \Gamma}{\partial \lambda_F}(\lambda_F, \lambda_L) = -\frac{\lambda_L}{1-\lambda_L} \leq 0$ implying $\frac{\partial Q^S}{\partial \lambda_F} \geq 0$, with strict inequalities if $\lambda_L > 0$. Similarly for λ_L , $\frac{\partial \Gamma}{\partial \lambda_L}(\lambda_F, \lambda_L) = \frac{1-\lambda_F}{(1-\lambda_L)^2} > 0$ implying $\frac{\partial Q^S}{\partial \lambda_L} < 0$. The effect of internalization if the follower is inactive follows directly from $\bar{q}'(\lambda_F) < 0$ and $\bar{q}(\lambda_F)$'s independence from λ_L . In the $\delta = 0$ case, the relevant derivative for an active follower is $-\frac{\Gamma'}{2(1+\Gamma)^2}$ and the result follows similarly. $\square$

An increase in leader internalization softens the leader's output choice all else equal, which because the follower's reaction function slope is less than one in magnitude unambiguously implies that total output decreases. The more surprising part of the proposition is the positive effect of follower internalization on equilibrium output with an active follower. This increase in equilibrium output requires that the leader's output response to the follower's softer behavior be so aggressive as to more than offset the drop in follower output. To see how greater follower internalization can trigger such an aggressive leader response, it is useful to rewrite the leader's first-order condition (7) in Lerner index form as

$$\frac{P(Q^S) - C}{P(Q^S)} = \frac{1}{\frac{1+\lambda_L \frac{\partial q_F^*}{\partial q_L}(q_L^*, \lambda_F)}{q_L^* + \lambda_L q_F^*(q_L^*, \lambda_F)} \frac{P(Q^S)}{-P'(Q^S)}}, \quad (10)$$

where the right-hand side is the inverse of the leader's perceived demand elasticity, which we hereafter denote by $\varepsilon_L(Q^S, \lambda_F, \lambda_L)$. Intuitively, the leader's response is sufficiently

aggressive as to raise equilibrium output provided that follower internalization increases the magnitude of the perceived elasticity term. We show in the appendix that for a given total output level, this perceived elasticity is the product of a follower responsiveness measure and ordinary demand elasticity,

$$\varepsilon_L(Q, \lambda_F, \lambda_L) = \frac{1 - \lambda_L \frac{\partial \tilde{q}_F}{\partial Q}(Q, \lambda_F)}{1 - \lambda_L \frac{\tilde{q}_F(Q, \lambda_F)}{Q}} \varepsilon(Q). \quad (11)$$

where $\tilde{q}_F(Q, \lambda_F)$ is the follower's inclusive best reply⁴ and $\varepsilon(Q) = -\frac{P'(Q)}{P(Q)Q}$. Provided that the leader internalizes ($\lambda_L > 0$), greater follower internalization raises the leader's perceived elasticity and equilibrium output so long as the follower responsiveness decreases, which happens if the elasticity of the follower's inclusive best-reply $\frac{\partial \tilde{q}_F}{\partial Q}(Q, \lambda_F)$ decreases. This property, i.e. that increasing follower internalization dampens follower responsiveness around an equilibrium, holds more generally for twice-differentiable homogeneous product demands so long they satisfy the standard decreasing marginal revenue condition. We expect this effect to be all the more significant at lower demand curvature levels where the follower is relatively less responsive. The following proposition establishes that the intuition holds in our model, i.e. that the responsiveness of equilibrium output with respect to follower internalization increases with demand curvature.

Proposition 2. *In a Stackelberg equilibrium with overlapping ownership and an active follower, the elasticity of equilibrium output with respect to follower internalization is inversely related to the demand curvature parameter δ .*

⁴The inclusive best-reply (ibr) or fitting-in function, which is used in aggregative games (Anderson et al. 2020), denotes the optimal follower output consistent with a given level of total output. It is obtained from the follower first-order condition (5) by substituting in $Q = q_L + q_F^*$.

Proof. Differentiating (8) and then simplifying,

$$\begin{aligned}
\varepsilon_{Q^S/\lambda_F} &= \frac{\partial \ln Q^S}{\partial \ln \lambda_F}(\lambda_F, \lambda_L) = -\frac{1}{1+\delta} \frac{\frac{1+\delta}{2} \left(-1 + \frac{2+\Gamma+2\delta}{\sqrt{(2+\Gamma)^2+4\delta\Gamma}} \right)}{-1 + \frac{1+\delta}{2} \left(-\Gamma + \sqrt{(2+\Gamma)^2+4\delta\Gamma} \right)} \left(-\frac{\lambda_L \lambda_F}{1-\lambda_L} \right) \\
&= \frac{2+\Gamma+2\delta - \sqrt{(2+\Gamma)^2+4\delta\Gamma}}{\sqrt{(2+\Gamma)^2+4\delta\Gamma} \left((1+\delta)\sqrt{(2+\Gamma)^2+4\delta\Gamma} - 2 - (1+\delta)\Gamma \right)} \frac{\lambda_L \lambda_F}{1-\lambda_L} \\
&= \frac{2}{\sqrt{(2+\Gamma)^2+4\delta\Gamma} \left(\Gamma + \sqrt{(2+\Gamma)^2+4\delta\Gamma} \right)} \frac{\lambda_L \lambda_F}{1-\lambda_L},
\end{aligned} \tag{12}$$

which is decreasing in δ . $\square$

From a normative perspective Proposition 1 implies that leader internalization is unambiguously anticompetitive, as is the case in other standard oligopoly models. Provided that the follower is active however, the proposition also establishes that follower internalization is procompetitive, in marked contrast with the intuitions drawn from other standard oligopoly models. Proposition 2 establishes that this effect of follower internalization is stronger, all else equal, in markets with a lower value of the demand curvature parameter.

To draw sharper policy implications for industries with overlapping ownership structures, we relate the leader and follower internalization levels back to the ownership model in Section 2. Proposition 1 describes the effects of follower and leader internalization, which in turn depend on the cross-ownership stakes s_F and s_L and the common ownership parameter λ in our ownership model. Assuming there is some common ownership for example, an increase in the follower's cross-ownership stake has a positive effect on output through two complementary channels, as it raises the follower's internalization level but also decreases the leader's internalization level. The overall effect is neatly captured by the parameter $\Gamma(\lambda_F, \lambda_L)$. Substituting for λ_F and λ_L gives $\Gamma(s_F, s_L, \lambda) = \frac{(1-s_F s_L)(1+\lambda)}{1-s_L+s_L(1-s_F)\lambda}$,

which is monotonic in cross-ownership shares and the common ownership level. Differentiating establishes the following effects of the ownership structure primitives on equilibrium output, consumer surplus, and welfare:

- Increasing in the follower cross-ownership stake s_F ,
- Decreasing in the leader cross-ownership stake s_L ,
- Increasing (decreasing) in the common ownership level λ if $s_L(2 - s_F) < (>)1$.

From a policy perspective, our results therefore imply that so long as the follower remains active, the acquisition of a noncontrolling stake in a leader by a follower firm is socially beneficial, as is an increase in symmetric common ownership provided that the leader's cross-ownership stake is not too large.

3.2 Entry deterrence and second mechanism

We now incorporate a positive entry cost into the model and study the leader's decision to deter or accommodate the follower. The leader can therefore be viewed as an incumbent who has sunk the entry cost and commits to an output level, and the follower is a potential entrant who has not yet paid the entry cost. These conditions can be understood as introducing an intermediate entry decision stage into the game, so the timing becomes (1) the leader determines its output, (2) the follower decides whether or not to enter, and (3) conditional upon entering the follower determines its output. We denote the entry cost by K and assume that $K < \pi^M$ so entry is viable where $\pi^M = B(1 + \delta) \left(\frac{\tilde{A}}{2 + \delta} \right)^{\frac{2 + \delta}{1 + \delta}}$ denotes monopoly profit.

We reason by backward induction and again start by analyzing the follower's problem. Conditional upon entering, at the output stage the follower faces the same optimization

problem as in the preceding section. This results in a positive follower output described by (5) if the leader's output is lower than $\bar{q}_L$ and zero otherwise.

Next we turn to the follower's entry decision. Conditional upon being active post-entry, the follower enters if

$$\pi_F(q_F^*(q_L, \lambda_F), q_L) + \lambda_F \pi_L(q_F^*(q_L, \lambda_F), q_L) > K + \lambda_F \pi_L(0, q_L). \quad (13)$$

Along with its effect on perceived profits at the output stage, internalization of the incumbent also increases the follower's opportunity cost of entry. Increasing the level of follower internalization has two effects on the follower's entry condition. On the left-hand side of (13), the envelope theorem establishes that an increase in follower internalization has only a direct effect on perceived profit, so $\frac{d[\pi_F + \lambda_F \pi_L]}{d\lambda_F}(q_F^*(q_L, \lambda_F), q_L) = \frac{\partial[\pi_F + \lambda_F \pi_L]}{\partial \lambda_F}(q_F^*(q_L, \lambda_F), q_L) = (P(q_F^*(q_L, \lambda_F) + q_L) - C)q_L$, which is positive. On the right-hand side, greater follower internalization raises the perceived opportunity cost of entering by the perceived marginal impact on incumbent or leader profit $\pi_L(0, q_L) = (P(q_L) - C)q_L$. The latter effect dominates so that greater follower internalization softens the follower's entry decision, making deterrence easier for the leader to achieve.

We next argue that there is a unique level of leader output $q_L^D(\lambda_F) \leq \bar{q}(\lambda_F)$ at which entry is deterred that is defined by the follower first-order condition (5) and entry condition (13), and which is a decreasing function of follower internalization. The follower's entry condition holds at $q_L = 0$ due to the restriction that K is lower than monopoly profit, and it is trivially violated if $q_L = \bar{q}_F(\lambda_F)$ in which case $q_F^* = 0$. Subtracting the right-hand side of (13) from the left-hand side to get the follower's entry incentive and differentiating with respect to q_L , using the envelope theorem again to cancel $q_F(q_L, \lambda_F)$ effects, the

marginal effect of leader output on the follower's entry incentive is

$$\frac{\partial \pi_F}{\partial q_L}(q_F^*, q_L) - \lambda_F \left(\frac{\partial \pi_L}{\partial q_L}(0, q_L) - \frac{\partial \pi_L}{\partial q_L}(q_F^*, q_L) \right). \quad (14)$$

The first term is negative, and the expression inside the brackets is positive because marginal revenue is decreasing with our demand specification. The follower's entry incentive is therefore decreasing in q_L , so by continuity there exists a single crossing point $q_L^D \in (0, \bar{q}_L(\lambda_F))$ that equates the two sides of (13) and at which the follower is deterred.

The demand specification (1) produces a semi-closed expression for $q_L^D(\lambda_F)$, which is found by using the follower's first-order condition (5) to get $q_L^D(Q^D) = \frac{1}{1-\lambda_F} \left(Q^D - \frac{\tilde{A} - (Q^D)^{1+\delta}}{(1+\delta)(Q^D)^\delta} \right)$ where $Q^D = q_F^*(q_L^D, \lambda_F) + q_L^D$ is total output if the follower reacts optimally, obtained by equating the two sides of (13).⁵ In the specific case of linear demand ($\delta = 0$) the deterrence output has an explicit form $q_L^D(\lambda_F) = \frac{\tilde{A} - 2\sqrt{K/B}}{1+\lambda_F}$. In all cases, it is straightforward to check that $q_L^D(\lambda_F)$ is a decreasing function of λ_F by an envelope theorem argument. Differentiating the follower's entry incentive with respect to λ_F gives simply the direct effect $-(\pi_L(0, q_L^D) - \pi_L(q_F^*, q_L^D)) < 0$, which implies $q_L^{D'}(\lambda_F) < 0$ given that the follower's entry incentive is decreasing in q_L .

Next we turn to the leader's problem. The possibility of deterring the follower implies that the leader's profit function is defined piecewise depending on whether or not the leader's output drives the follower to enter. The leader's payoff can accordingly have one or two local optima, corresponding to accommodation or deterrence strategies. The accommodation strategy involves both firms being active. It is characterized by an optimal leader output $q_L^A(\lambda_F, \lambda_L)$ corresponding to the optimal leader output in Stackelberg

⁵That is, $Q^D \in [q^M, \bar{q}(\lambda_F))$ solves $\frac{(\tilde{A} - Q^{1+\delta})^2}{(1+\delta)Q^\delta} - \frac{K}{B} = \lambda_F(\tilde{A} - (q_L^D(Q^D))^{1+\delta})q_L^D(Q^D)$ where $q_L^D(Q^D)$ is as defined in the text.

equilibrium.⁶ Under deterrence, the leader chooses the larger of $q_L^D(\lambda_F)$ and q^M and operates as the only firm in the market. If $q_L^D(\lambda_F) \leq q^M$, the leader is said to block the follower naturally by producing the monopoly output. This situation arises if either the entry cost or follower internalization is high enough, and in either case the leader's choice of strategy is trivial. Otherwise if $q_L^D(\lambda_F) > q^M$ the leader's strategy is determined by comparing the respective profits from accommodation and from deterrence, $\pi^A(\lambda_F, \lambda_L) = \pi_L(q_F^*(q_L^A, \lambda_F), q_L^A) + \lambda_L(\pi_F(q_F^*(q_L^A, \lambda_F), q_L^A) - K)$ and $\pi^D(\lambda_F) = \pi_L(0, q_L^D)$, where we omit the arguments of leader output when there is no risk of confusion. The difference between the two profit levels $\mathcal{D}(\lambda_F, \lambda_L) = \pi^A(\lambda_F, \lambda_L) - \pi^D(\lambda_F)$ exhibits complex behavior with respect to follower internalization, even with the demand specification (1). We therefore have the following partial description of how the leader's strategy relates to leader and follower internalization.

Proposition 3. *Suppose that the leader accommodates if the firms are independent, i.e. at $(\lambda_F, \lambda_L) = (0, 0)$. Then in the space of internalization levels $(\lambda_F, \lambda_L) \in [0, 1]^2$, the equilibrium involves accommodation on the left edge $\{(0, \lambda_L) : \lambda_L \in [0, 1]\}$ and deterrence on the right edge $\{(1, \lambda_L) : \lambda_L \in [0, 1]\}$.*

Proof. For the accommodation part it is sufficient to show that $\frac{\partial \mathcal{D}}{\partial \lambda_L}(0, \lambda_L) = \frac{\partial \pi_L^A}{\partial \lambda_L}(0, \lambda_L) > 0$. Due to the envelope theorem, changes in λ_L only have a direct effect on leader profit, $(P(Q^S) - C)\tilde{q}_F(Q^S, \lambda_F) - K$. Because the inequality (13) holds under accommodation, $(P(Q^S) - C)(\tilde{q}_F(Q^S, \lambda_F) + \lambda_F q_L^A) > K + \lambda_F(P(q_L^A) - C)q_L^A$. Therefore, $(P(Q^S) - C)\tilde{q}_F(Q^S, \lambda_F) - K > \lambda_F(P(q_L^A) - P(Q^S))q_L^A$, which establishes that the derivative has the desired sign. For the deterrence part consider internalization levels with $\lambda_F = 1$. Then the follower maximizes joint profit, so it sets an output $q_F^* = q^M - q_L$ whenever it is active.

⁶To obtain the exact expression in our model in terms of the total output expression Q^S , substitute $Q^S = q_F^* + q_L^A$ into the follower first-order condition (5) and rearrange to get $q_L^A(\lambda_F, \lambda_L) = \frac{(2+\delta)Q^{S^{1+\delta}} - \tilde{A}}{(1+\delta)(1-\lambda_F)Q^{S^\delta}}$.

The leader naturally deters the follower in this case by producing the monopoly output, with the added perceived benefit of economizing on the follower’s perceived entry cost if $\lambda_L > 0$. $\square$

Though this is not central to the rest of our analysis, Proposition 3 has an interesting implication with respect to the leader’s behavior at high internalization levels. As internalization levels approach the top left corner $(\lambda_F, \lambda_L) = (0, 1)$, the leader’s accommodation output converges to zero. Eventually the leader opts to avoid producing entirely despite having a first-mover advantage. In the limit, the leader therefore surrenders the market to the follower entirely, with accommodation resulting in an unusual “follower monopoly” situation.

We are primarily interested though in how internalization acts on the leader’s margin between accommodation and deterrence, because the qualitative change from accommodation to deterrence reflects more aggressive leader behavior with the potential to raise output and eventually welfare (which must also account for follower entry cost in this setup). The equilibrium characterization in Proposition 3 suggests that higher follower internalization, and eventually higher follower and leader internalization, are ultimately associated with deterrence. In the context of our stylized ownership model, this means that a high enough follower cross-ownership stake or high enough common ownership drive the leader to opt for deterrence.

Around the accommodation-deterrence boundary where the leader is indifferent between both strategies, local internalization changes that shift the equilibrium toward deterrence generally have two positive effects on welfare. In the first place, the industry and society economize on the follower’s entry cost with deterrence. But in addition to this mechanical increase in welfare, total output also jumps up which also raises welfare for the reasons seen in the preceding section. Beyond the jump, further increases in internaliza-

tion (or in follower internalization, as the leader's deterrence output is independent of its internalization level) subsequently lower total output and welfare. Nevertheless the local behavior at the boundary between accommodation and deterrence opens the possibility for welfare to be higher with internalization than at lower internalization levels where the leader's strategy has not switched, and in particular to be higher than the welfare realized in a benchmark equilibrium with independent firms.

To establish that internalization can provoke such a welfare-increasing shift in the equilibrium regime, we focus on linear demand and symmetric internalization for which we can provide a direct argument. Symmetric internalization may reflect identical cross-ownership stakes, but is also often used as a first approximation to common ownership effects. Numerical evaluations, which we return to further below, suggest that the pattern for other demand curvature values and asymmetric follower and leader internalization levels is similar to what we describe here. The following proposition formalizes the notion of a welfare-increasing equilibrium shift due to overlapping ownership.

Proposition 4. *Suppose internalization levels are symmetric ($\lambda_F = \lambda_L = \lambda$) and demand is linear ($\delta = 0$). If the equilibrium with independent firms (i.e. with $\lambda = 0$) involves accommodation, then there exist positive internalization levels $\lambda^* > 0$ where the leader deters and with higher welfare, i.e. such that $W(\lambda^*, \lambda^*) > W(0, 0)$.*

Proof. Assume that $K < \frac{\tilde{A}^2 B}{32}(3 - 2\sqrt{2})$ so the independent firm equilibrium involves accommodation. In this accommodation equilibrium, evaluating total output using (9) gives $Q^S(0, 0) = \frac{3}{4}\tilde{A}$. Total welfare consists of a flow welfare term, whose expression is given in footnote 3 above, net of the follower's entry cost. Therefore,

$$W(0, 0) = BQ^S(0, 0)(\tilde{A} - \frac{1}{2}Q^S(0, 0)) - K = \frac{15}{32}\tilde{A}^2 B - K. \quad (15)$$

With symmetric internalization, we determine the equilibrium as follows. Start with deterrence, where total output is just $q_L^D(\lambda) = \frac{\tilde{A} - 2\sqrt{K/B}}{1+\lambda}$. The leader's deterrence profit is therefore $\pi_L^D(\lambda) = (P(q_L^D(\lambda)) - C)q_L^D(\lambda) = \frac{B}{(1+\lambda)^2} \left(\tilde{A} - 2\sqrt{\frac{K}{B}} \right) \left(\lambda\tilde{A} + 2\sqrt{\frac{K}{B}} \right)$.

Next we evaluate the leader's accommodation profit. Taking $q_L^A(\lambda, \lambda) = \frac{2Q^S - \tilde{A}}{1-\lambda}$ (see footnote 6 above) and $Q^S(\lambda, \lambda) = \frac{3+\lambda}{2(2+\lambda)}\tilde{A}$ from (9) gives $\pi_L^A(\lambda, \lambda) = B(\tilde{A} - Q^S)(\lambda Q^S + (1-\lambda)q_L^A) - \lambda K = \frac{\tilde{A}^2 B(1+\lambda)^2}{4(2+\lambda)} - \lambda K$.

The leader shifts from accommodation to deterrence at the boundary $\mathcal{D}(\lambda, \lambda) = \pi_L^A(\lambda, \lambda) - \pi_L^D(\lambda) = 0$. Deterrence occurs if the profit difference is weakly negative, i.e. after normalizing by $\tilde{A}^2 B$

$$\frac{(1+\lambda)^2}{4(2+\lambda)} - \lambda \frac{K}{\tilde{A}^2 B} - \frac{1}{(1+\lambda)^2} \left(1 - 2\sqrt{\frac{K}{\tilde{A}^2 B}} \right) \left(\lambda + 2\sqrt{\frac{K}{\tilde{A}^2 B}} \right) \leq 0. \quad (16)$$

This inequality can be expressed equivalently as

$$(\lambda^2 + 3\lambda + 4) \frac{K}{\tilde{A}^2 B} - 2\sqrt{\frac{K}{\tilde{A}^2 B}} + \frac{(1 - 2\lambda - \lambda^2)^2}{4(1-\lambda)(2+\lambda)} \leq 0. \quad (17)$$

Solving for the lower root, the leader shifts from accommodation to deterrence for entry costs satisfying $K \geq \left(\frac{1 - \frac{1+\lambda}{2} \sqrt{\frac{(1+\lambda)(4-3\lambda-4\lambda^2-\lambda^3)}{2-\lambda-\lambda^2}}}{4+3\lambda+\lambda^2} \right)^2 \tilde{A}^2 B$, over the range of admissible entry costs $\left[0, \frac{\tilde{A}^2 B}{32} (3 - 2\sqrt{2}) \right)$.

Because the follower does not enter, welfare under deterrence just consists of the flow welfare expression

$$W(\lambda, \lambda) = Bq_L^D(\lambda) \left(\tilde{A} - \frac{1}{2}q_L^D(\lambda) \right) = B \frac{\tilde{A} - 2\sqrt{\frac{K}{B}}}{1+\lambda} \left(\tilde{A} - \frac{1}{2} \frac{\tilde{A} - 2\sqrt{\frac{K}{B}}}{1+\lambda} \right). \quad (18)$$

The welfare difference between deterrence with internalization and the accommodation

outcome with independent firms, (18–15), is positive if

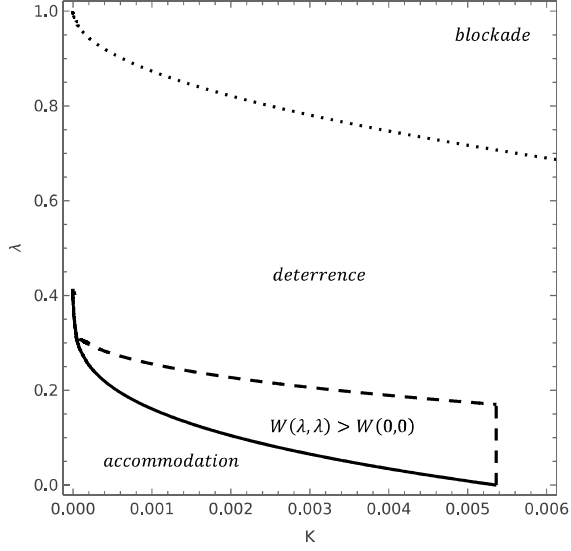
$$-(1 - 2\lambda - \lambda^2)\frac{K}{\tilde{A}^2 B} - 2\lambda\sqrt{\frac{K}{\tilde{A}^2 B}} + \frac{1 + 2\lambda - 15\lambda^2}{32} > 0. \quad (19)$$

Provided that $\lambda < \sqrt{2}-1$, the above inequality is satisfied if $K < \left(\frac{2\lambda + \frac{1+\lambda}{2\sqrt{2}}\sqrt{1-2\lambda+15\lambda^2}}{1-2\lambda-\lambda^2}\right)^2 \tilde{A}^2 B$.

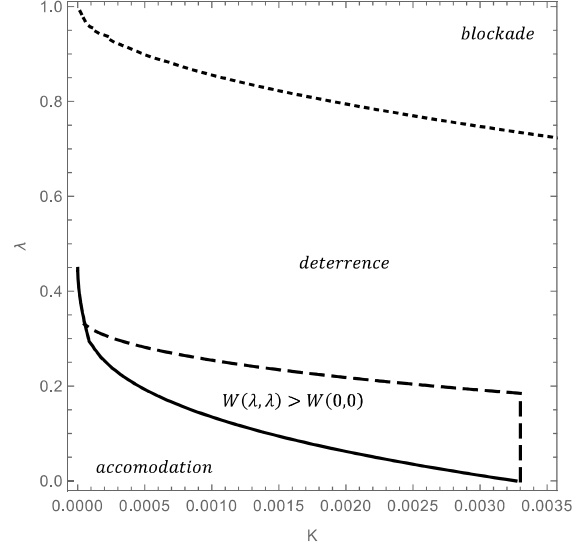
It is then straightforward to check that the conditions (17) for deterrence at $\lambda > 0$ and (19) for higher welfare than in the independent firm accommodation outcome are jointly met for a range of entry costs at specific internalization levels $\lambda^* \in (0, \sqrt{2}-1)$. For example, for $\lambda = 0.2$, (17) holds for $\frac{K}{\tilde{A}^2 B} \geq 5.6 \times 10^{-4}$ while (19) holds if $\frac{K}{\tilde{A}^2 B} \leq 2.59$, which is weaker than the initial constraint that independent firms accommodate, $\frac{K}{\tilde{A}^2 B} \leq 5.4 \times 10^{-3}$. $\square$

Proposition 4 establishes the existence of positive internalization levels where welfare is higher than with fully independent firms. The possibility of internalization generating such a welfare-increasing equilibrium shift is relevant to regulatory discussions surrounding common ownership holdings, as it shows internalization can have positive economic consequences in a very standard industry setting without either R&D spillovers or vertical externalities. The symmetric internalization assumption is not so restrictive in this regard, as it is consistent with a widespread view of common ownership as having fairly uniform effects as a first approximation.

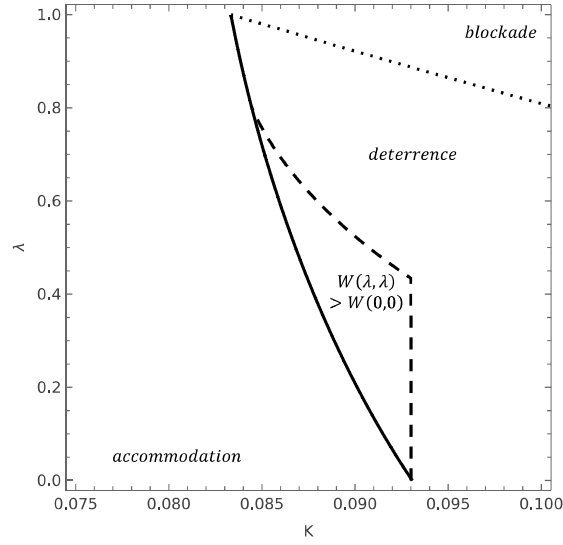
The Stackelberg equilibrium solution (8) and semi-closed form $q_L^D(Q^D)$ allow us to pursue the analysis further using numerical methods. The equilibrium and welfare pattern varies smoothly as the symmetric internalization assumption is relaxed, yielding similar predictions with respect to increases in follower internalization. Allowing demand curvature to vary reveals similar equilibrium patterns and welfare wedges, which are illustrated in Figure 1. Comparing the patterns with linear (panel 1a) and more convex (panel



(a) homogeneous goods ($\delta = 0$)



(b) homogeneous goods ($\delta = -0.5$)



(c) differentiated goods and price competition ($\delta = 0, \theta = 0.5$)

Figure 1: Accommodation-deterrence (solid) and deterrence-blockade (dotted) boundaries in (K, λ) -space with symmetric internalization ($\lambda_F = \lambda_L = \lambda$). Dashed curves delimit higher welfare under deterrence with internalization than accommodation with independent firms. (1a), (1b): homogeneous goods and quantity competition; (1c): price competition with differentiated products (see Section 4.2).

1b) demands suggests that the positive welfare effect is slightly larger with more convex demand. That is to say, for comparable fixed cost levels (i.e. as shares of the highest fixed cost consistent with accommodation), the positive welfare effect extends to slightly higher internalization levels with the more convex demand specification, which as we saw in the preceding section fosters more aggressive leader behavior. The numerical analysis suggests that output effects are secondary with these demand specifications, as most of the welfare gain seems to be driven by economies on the follower's entry cost evidenced by the growing height of the wedge moving rightward. The last panel in the figure shows a similar pattern in the empirically relevant case of price competition with differentiated products, which we develop along with other extensions of our model in the next section.

4 Discussion and extensions

We next clarify the scope of our procompetitive results by developing several extensions of the model. These extensions are all based on the linear demand specification. In the first extension, we examine quantity competition with product differentiation and find that lower product substitutability dampens both the leader's aggressiveness and its ability to deter. The second extension examines price competition with differentiation, and shows that a welfare-increasing shift to deterrence can occur, unlike under price competition with homogeneous goods. The third extension involves a hierarchy of firms making sequential output decisions, and we find the same procompetitive effect of backward internalization by follower firms as in the duopoly case.

4.1 Quantity competition with product differentiation

Suppose that the products of the two firms are differentiated, with each firm's inverse demand given by

$$P_i = A - B(q_i + \theta q_j), \quad i, j \in \{F, L\}, \quad i \neq j \quad (20)$$

where $A, B > 0$ and $C \in [0, A)$ are constants and $|\theta| < 1$ is a differentiation parameter.⁷ We set $\tilde{A} = \frac{A-C}{B}$ as above.

We are primarily interested in situations similar to our baseline model where overlapping ownership raises welfare by inducing more aggressive leader behavior. We therefore start in this section by maintaining the assumption that firms compete in outputs rather than prices, and focus the discussion on substitute products.

We start by deriving the Stackelberg equilibrium under internalization. Proceeding by backward induction, solving the follower's problem (4) yields its best-reply

$$q_F^*(q_L, \lambda_F) = \max \left\{ 0, \frac{\tilde{A} - \theta(1 + \lambda_F)q_L}{2} \right\}. \quad (21)$$

Note that here, by producing $\bar{q}(\lambda_F) = \frac{\tilde{A}}{\theta(1+\lambda_F)}$, the leader can profitably shut the follower out but only if the products are sufficiently close substitutes so that $\theta > \frac{1}{1+\lambda_F}$.

Turning next to the leader's problem, solving (6) yields the first-order condition

$$\tilde{A} - 2q_L^* - \theta(1 + \lambda_L)q_F^*(q_L^*, \lambda_F) - \theta(1 - \lambda_F\lambda_L)q_L^* \frac{\partial q_F^*}{\partial q_L}(q_L^*, \lambda_F) = 0, \quad (22)$$

⁷This demand specification arises if there is a unit mass of consumers with quasi-linear utility $U(q_F, q_L, Y) = A(q_F + q_L) - \frac{B}{2}(q_F^2 + q_L^2 + 2\theta q_F q_L) + Y$.

which defines equilibrium outputs

$$\begin{aligned} q_F^S(\lambda_F, \lambda_L) &= \frac{1}{2}(\tilde{A} - \theta(1 + \lambda_F)q_L^S(\lambda_F, \lambda_L)), \\ q_L^S(\lambda_F, \lambda_L) &= \frac{2 - \theta(1 + \lambda_L)}{4 - \theta^2(1 + \lambda_F)(2 + \lambda_L - \lambda_F\lambda_L)}\tilde{A}, \end{aligned} \quad (23)$$

if $\theta(1 + \lambda_F)(2 + \theta(1 - \lambda_F\lambda_L)) < 4$ so both firms are active, and $q_F^S(\lambda_F, \lambda_L) = 0$, $q_L^S(\lambda_F, \lambda_L) = \bar{q}_L(\lambda_F)$ otherwise.

To determine how internalization affects an interior equilibrium where both firms are active, start with the partial derivatives of equilibrium leader output

$$\frac{\partial q_L^S}{\partial \lambda_F}(\lambda_F, \lambda_L) = \frac{2\theta^2(1 - \lambda_F\lambda_L)}{4 - \theta^2(1 + \lambda_F)(2 + \lambda_L - \lambda_F\lambda_L)}q_L^S(\lambda_F, \lambda_L) \quad (24)$$

and

$$\frac{\partial q_L^S}{\partial \lambda_L}(\lambda_F, \lambda_L) = -\theta \left(\frac{1}{2 - \theta(1 + \lambda_L)} - \frac{\theta(1 - \lambda_F^2)}{4 - \theta^2(1 + \lambda_F)(2 + \lambda_L - \lambda_F\lambda_L)} \right) q_L^S(\lambda_F, \lambda_L). \quad (25)$$

The first partial derivative (24) is positive, meaning that greater follower internalization raises leader output regardless of whether the products are complements or substitutes. Because the bracketed term in (25) is positive in an interior equilibrium, the second partial derivative has the opposite sign of θ , meaning that increased leader internalization lowers the leader's output with substitute products and increases it with complementary products.

Next, the partials for equilibrium follower output are

$$\begin{aligned}\frac{\partial q_F^S}{\partial \lambda_F}(\lambda_F, \lambda_L) &= -\frac{\theta}{2} q_L^S(\lambda_F, \lambda_L) \left(1 + (1 + \lambda_F) \frac{\frac{\partial q_L^S}{\partial \lambda_F}(\lambda_F, \lambda_L)}{q_L^S(\lambda_F, \lambda_L)} \right) \\ &= -\frac{\theta}{2} q_L^S(\lambda_F, \lambda_L) \frac{4 - \theta^2 \lambda_F (1 + \lambda_F) (1 + \lambda_L)}{4 - \theta^2 (1 + \lambda_F) (2 + \lambda_L - \lambda_F \lambda_L)},\end{aligned}\quad (26)$$

which has the opposite sign of θ meaning that the follower's equilibrium output decreases with substitute products and increases with complements, and

$$\frac{\partial q_F^S}{\partial \lambda_L}(\lambda_F, \lambda_L) = -\frac{\theta(1 + \lambda_F)}{2} \frac{\partial q_L^S}{\partial \lambda_L}(\lambda_F, \lambda_L) \quad (27)$$

which is positive for all θ . To summarize:

Proposition 5. *Under sequential output choices and with the linear differentiated product demand specification (20),*

- (i) *Follower and leader internalization have positive cross-effects on individual outputs, i.e. $\frac{\partial q_L^S}{\partial \lambda_F}(\lambda_F, \lambda_L), \frac{\partial q_F^S}{\partial \lambda_L}(\lambda_F, \lambda_L) > 0$;*
- (ii) *Follower and leader internalization have negative (positive) own effects on individual outputs if the products are substitutes (complements), i.e. $\frac{\partial q_F^S}{\partial \lambda_F}(\lambda_F, \lambda_L), \frac{\partial q_L^S}{\partial \lambda_L}(\lambda_F, \lambda_L) < (>)$ 0 if $\theta > (<) 0$.*

With pure cross-ownership the output effects of rival stakes are the same as in Proposition 5, but with positive common ownership ($\lambda > 0$) they are more involved. In that case an increase in the follower cross-ownership share s_F for example, which raises λ_F and lowers λ_L , unambiguously increases the leader's output and lowers the follower's output if products are substitutes. With complementary products however the leader and follower internalization effects have opposing signs.

With homogeneous products, we had found that welfare increases because of the leader's aggressive output response to follower internalization. With differentiation, total output is no longer relevant but we can get some insight based on the leader's output $q_L^S(\lambda_F, \lambda_L)$.⁸ A finding in the homogeneous goods case was the role of demand curvature in governing how equilibrium output responds to follower internalization, as described in Proposition 2. We can proceed similarly here by examining how the leader's output response varies with product substitutability and obtain a similar result. From (24), the elasticity of the leader's equilibrium output with respect to follower internalization is

$$\varepsilon_{q_L^S/\lambda_F} = \frac{\partial \ln q_L^S}{\partial \ln \lambda_F}(\lambda_F, \lambda_L) = \frac{2\theta^2(1 - \lambda_F\lambda_L)\lambda_F}{4 - \theta^2(1 + \lambda_F)(2 + \lambda_L - \lambda_F\lambda_L)} \quad (28)$$

which is clearly increasing in θ if products are substitutes. We conclude therefore that greater product differentiation (lower θ) attenuates the aggressiveness of the leader's response to follower internalization. The procompetitive effect of follower internalization is therefore more important in markets where products are closer substitutes.

We next turn to the second mechanism in our homogeneous goods analysis, involving a welfare-enhancing shift to deterrence. Starting from an interior point on the follower's best response (21), the follower's zero profit condition with product differentiation is

$$B(\tilde{A} - q_F^* - \theta q_L)q_F^* + \lambda_F B(\tilde{A} - \theta q_F^* - q_L)q_L = K + \lambda_F B(\tilde{A} - q_L)q_L. \quad (29)$$

⁸Total welfare has the semi-closed in terms of leader output $W(\lambda_F, \lambda_L) = \frac{3}{8}\tilde{A}^2 + \frac{4 - \theta(3 + \lambda_F)}{4}\tilde{A}q_L(\lambda_F, \lambda_L) - \frac{4 - \theta^2(1 + \lambda_F)(5 + 4\lambda_F)}{8}q_L^2(\lambda_F, \lambda_L)$ that limits further analytic results, even if a lower leader output response presumably lowers welfare.

Substituting for $q_F^S(\lambda_F, \lambda_L)$ and solving for the deterrence output yields

$$q_L^D(\lambda_F) = \frac{\tilde{A} - 2\sqrt{\frac{K}{B}}}{\theta(1 + \lambda_F)}. \quad (30)$$

Deterrence is only relevant if products are substitutes. When this is the case, the deterrence output is decreasing in the substitution parameter. As may be expected therefore if the submarkets that the firms operate in are more separated, allowing for imperfectly substitutable products also dampens the second positive welfare effect identified in the main model.

4.2 Deterrence under price competition with differentiation

With price competition, softer follower behavior leads to higher leader prices rather than more aggressive behavior, so we do not pursue the first procompetitive mechanism further here. However there can be welfare-increasing effects with respect to our second procompetitive mechanism involving deterrence. This is all the more relevant as much of the empirical evidence surrounding common ownership, like airline pricing, has involved industries plausibly characterized by price competition with differentiation. Under simplified assumptions, we show below that follower internalization can cause a welfare-increasing shift to deterrence, which would not be possible with homogeneous goods.

To start with, invert the demand system (20) to obtain

$$q_i(P_i, P_j) = \hat{A} - \hat{B}P_i + \theta\hat{B}P_j, \quad i, j \in \{F, L\}, \quad i \neq j \quad (31)$$

where $\hat{A} = \frac{A}{(1+\theta)B}$ and $\hat{B} = \frac{1}{(1-\theta^2)B}$. We suppose for simplicity that costs are zero and that internalization levels are symmetric, as in our analysis of the homogeneous case.

Solving the follower's problem $\max_{P_F} [\pi_F(P_F, P_L) + \lambda \pi_L(P_F, P_L)]$ yields the reaction function

$$P_F^*(P_L, \lambda) = \frac{1}{2} ((1 - \theta)A + \theta(1 + \lambda)P_L). \quad (32)$$

$P_F^*(P_L, \lambda)$ is increasing in λ , as internalization softens the follower's behavior. Substituting back into $\pi_F(P_F, P_L) = (\hat{A} - \hat{B}P_F + \theta\hat{B}P_L)P_F + \lambda(\hat{A} + \theta\hat{B}P_F - \hat{B}P_L)P_L$ gives the follower's optimal perceived profit from entering

$$\pi_F(P_F^*(P_L), P_L) = \frac{\hat{B}}{4} ((\theta^2(1 + \lambda)^2 - 4\lambda)P_L^2 + 2A(1 - \theta)(\theta + 2\lambda + \theta\lambda)P_L + A^2(1 - \theta)^2) - K. \quad (33)$$

If the follower does not enter, the leader faces demand $q_L(P_L) = \frac{A - P_L}{B}$ and the follower's perceived profit is just $\lambda \frac{(A - P_L)P_L}{B}$. The follower's net profit from entry is therefore a quadratic in P_L .⁹ Setting the follower's net profit to zero yields a straightforward solution for the deterrence price,

$$P_L^D(\lambda) = \frac{1 - \theta}{\theta(1 - \lambda)} \left(-A + 2\sqrt{\frac{1 + \theta}{1 - \theta}BK} \right). \quad (34)$$

With perfect substitutes, the deterrence price would be zero and deterrence would not occur. Substituting into $\frac{(A - P_L^D)P_L^D}{B}$ gives the leader's deterrence profit,

$$\pi_L^D(\lambda) = \frac{1 - \theta}{\theta^2(1 - \lambda)^2 B} \left(-A + 2\sqrt{\frac{1 + \theta}{1 - \theta}BK} \right) \left(A(1 - \lambda\theta) - 2(1 - \theta)\sqrt{\frac{1 + \theta}{1 - \theta}BK} \right). \quad (35)$$

Under accommodation, the leader's payoff $\pi_L(P_F^*(P_L, \lambda), P_L) + \lambda \pi_F(P_F^*(P_L, \lambda), P_L)$ is

⁹The exact expression is $\frac{\theta^2(1 - \lambda)^2}{4(1 - \theta^2)B}P_L^2 + \frac{\theta(1 - \lambda)A}{2(1 + \theta)B}P_L + \frac{(1 - \theta)A^2}{4(1 + \theta)B} - K$.

a quadratic in P_L .¹⁰ Maximizing gives the leader's optimal price under accommodation,

$$P_L^A(\lambda) = \frac{(1 - \theta)(2 + \theta + \lambda\theta)A}{4 + \theta^2\lambda^3 - 3\theta^2\lambda - 2\theta^2}, \quad (36)$$

which is an increasing function of the internalization parameter. The resulting accommodation profit is

$$\pi_L^A(\lambda) = \frac{(1 - \theta)(1 + \lambda)(\theta^2(1 - \lambda)^2(1 + \lambda) + 4(1 + \theta))A^2}{4(1 + \theta)(4 - 2\theta^2 - 3\theta^2\lambda + \theta^2\lambda^3)B} - \lambda K. \quad (37)$$

At points where $P_L^D \geq \frac{A}{2B}$ so entry is not naturally blockaded, the leader opts for deterrence whenever $\pi_L^D \geq \pi_L^A$, i.e. when (35) is greater than (37). In normative terms, because costs are zero welfare is just given by the utility specification (see footnote 7), which gives $W^D(K, \lambda) = Aq_L^D(\lambda) - \frac{1}{2}Bq_L^2(\lambda) = \frac{A^2 - P_L^{D^2}(\lambda)}{2B}$ under deterrence and $W^A(K, \lambda) = A(q_F(P_L^A, P_F^A) + q_L(P_L^A, P_F^A)) - \frac{1}{2}B(q_F^2(P_L^A, P_F^A) + q_L^2(P_L^A, P_F^A) + 2\theta q_F(P_L^A, P_F^A)q_L(P_L^A, P_F^A)) - K$, where $q_F(P_L^A, P_F^A)$ and $q_L(P_L^A, P_F^A)$ are the Stackelberg equilibrium outputs given P_L^A and $P_F^A(P_L^A)$.

To see how our second mechanism arises under price competition with product differentiation, we set $A = B = 1$ and $\theta = 0.5$. Accommodation, deterrence, and blockade regions are plotted in panel (1c) of Figure 1 above. We plot these regions in (K, λ) -space over a narrower range of entry costs for visibility, as there is a lower range of entry costs where deterrence does not occur at any internalization level, in contrast with the other cases we studied. The dashed curve in the figure delimits where welfare is higher than under accommodation with independent firms, showing a similar pattern to the homogeneous goods cases. The downward-sloping accommodation-deterrence barrier establishes that internalization exercises a similar procompetitive effect as under quantity compe-

¹⁰The exact expression is $\frac{1}{4(1-\theta^2)B} ((-4 - \theta^2\lambda^3 + 3\theta^2\lambda + 2\theta^2)P_L^2 + 2(1 - \theta)(2 + \theta + \lambda\theta)AP_L + \lambda(1 - \theta)^2A^2)$.

tition with homogeneous goods. As conjectured in our main analysis, economizing the follower's entry cost is a significant driver of welfare here also, as the wedge increases with the entry cost.

4.3 Internalization with N firms

To determine to what extent positive internalization effects persist with more than two firms, we next consider an industry with a hierarchy of N firms that make sequential output choices. We maintain the linear specification which allows us to leverage the Stackelberg independence property described by [Hinnosaar \(2021\)](#) to obtain sharper results, and set costs to zero.

Suppose for simplicity that only a single firm, I , holds a stake in another firm, J , generating an internalization level λ_I^J . For any $Z \in \{1, \dots, N\}$, we hereafter use $\bar{Q}_Z = \sum_{i=1}^Z q_i$ to denote the cumulative output of the first Z firms. The following heuristic argument shows how the effect of internalization described in Proposition 1 carries over to an N -firm hierarchy, in the sense that as in the duopoly case, internalization by a single follower firm does not harm welfare.

Let $I > J$ so firm I internalizes the payoff of a predecessor firm in the sequence of output choices, a situation of *backward internalization*. Our main analysis for example had $(I, J) = (2, 1)$ with $\lambda_2^1 = \lambda_F$. Without loss of generality, we can set $I = N$.¹¹

¹¹Suppose that there are N active firms, with $I < N$. Proceeding by backward induction, the reactions of successor firms $I + 1, \dots, N$ in the sequence of output decisions and the resulting residual demands are as follows.

For firm N , $\max_{q_N} [(A - B\bar{Q}_{N-1} - Bq_N)q_N]$ yields the reaction $q_N^* = \frac{1}{2B}(A - B\bar{Q}_{N-1})$. The residual demand of the preceding firm, firm $N - 1$, is $A - B\bar{Q}_{N-1} - Bq_N^* = \frac{1}{2}(A - B\bar{Q}_{N-1})$.

Next suppose that the residual demands induced by all successors of a given firm $J \geq I$ have the same form, so in particular firm J 's residual demand is $\frac{1}{2^{N-J}}(A - B\bar{Q}_J)$. Firm J 's output choice, $\max_{q_J} [(A - B\bar{Q}_{J-1} - Bq_J)q_J]$, then yields the reaction $q_J^* = \frac{1}{2B}(A - B\bar{Q}_{J-1})$, and therefore a similar residual $\frac{1}{2^{N-J+1}}(A - B\bar{Q}_{J-1})$ for firm $J - 1$.

We conclude that the residual demand of any firm $I < N$ whose successors do not internalize is $\frac{1}{2^{N-I}}(A - B\bar{Q}_I)$. We can therefore subsume all subsequent output choices of firms which do not internalize

Under these assumptions, the output choice problem of the last firm is

$$\max_{q_N} [(A - B\bar{Q}_{N-1} - Bq_N)(\lambda_N^J q_J + q_N)], \quad (38)$$

which results in a reaction function $q_N^*(\lambda_N^J) = \max \{0, \frac{1}{2B} (A - B\bar{Q}_{N-1} - \lambda_N^J Bq_J)\}$. If the last firm is active, the residual demand for the $N-1^{\text{th}}$ firm is therefore $A - B\bar{Q}_{N-1} - Bq_N^* = \frac{1}{2B} (A - B\bar{Q}_{N-1} + \lambda_N^J Bq_J)$. The $N-1^{\text{th}}$ firm's decision problem is then

$$\max_{q_{N-1}} \left[\frac{1}{2} (A - B\bar{Q}_{N-2} - Bq_{N-1} + \lambda_N^J Bq_J) q_{N-1} \right], \quad (39)$$

which results in a reaction function $q_{N-1}^*(\lambda_N^J) = \max \{0, \frac{1}{2B} (A - B\bar{Q}_{N-2} + \lambda_N^J Bq_J)\}$ if firm $N-1$ is active, in which case the residual demand is $\frac{1}{4} (A - B\bar{Q}_{N-2} + \lambda_N^J Bq_J)$ for the preceding firm $N-2$.

Continuing by induction, in an equilibrium where all firms are active, firm J 's residual demand accounting for firm N 's backward internalization has the form $\frac{1}{2^{N-J}} (A - B\bar{Q}_J + \lambda_N^J Bq_J)$. Its decision problem is then

$$\max_{q_J} \left[\frac{1}{2^{N-J}} (A - B\bar{Q}_{J-1} - B(1 - \lambda_N^J) q_J) q_J \right], \quad (40)$$

so its optimal output is $q_J^*(\lambda_N^J) = \max \left\{ 0, \frac{1}{2B(1-\lambda_N^J)} (A - B\bar{Q}_{J-1}) \right\}$. If $J=1$, then define $\bar{Q}_{J-1} = 0$ and total output is entirely determined, with $q_1^*(\lambda_N^J) = \frac{A}{2B(1-\lambda_N^J)}$ for the first firm, $q_2^*(\lambda_N^J) = \frac{1}{2B} (A - Bq_1^*(\lambda_N^J) + \lambda_N^1 Bq_1^*(\lambda_N^J)) = \frac{A}{4B}$ for the second and succeeding firms, up to $q_N^*(\lambda_N^J) = \left(\sum_{i=1}^N \frac{1}{2^i} - \frac{\lambda_N^J}{2(1-\lambda_N^J)} \right) \frac{A}{B}$. If $J > 1$, observe that the residual demand for firm $J-1$ is $\frac{1}{2^{N-J}} (A - B\bar{Q}_{J-1} - B(1 - \lambda_N^J) q_J^*(\lambda_N^J)) = \frac{1}{2^{N-J+1}} (A - B\bar{Q}_{J-1})$, so all output choices by preceding firms are independent of the last firm's internalization, resulting in

by scaling the demands for firm I onward by 2^{N-I} , and proceed as if firm I were the last firm in the hierarchy.

standard Stackelberg outputs for the firms preceding J .

In line with the Stackelberg independence property, successor firms therefore offset the effect of subsequent backward internalization resulting in the same total output as in the Stackelberg equilibrium with independent firms. We therefore have established:

Proposition 6. *In the linear Stackelberg model with N active firms choosing output successively, total output and welfare are unaffected by backward internalization of a single firm.*

5 Concluding remarks

This paper incorporates overlapping ownership, with common and cross-ownership elements, into a sequential-move duopoly framework. In contrast with other standard oligopoly frameworks, internalization can lead to more competitive outcomes and higher welfare. The main insight driving our results is that softer follower behavior due to internalization can have procompetitive effects by triggering sufficiently more aggressive leader behavior. We identify two mechanisms through which such effects arise.

First, we show that increased follower internalization induces the leader to expand output enough in Stackelberg equilibrium that total output and welfare increase, with lower leader internalization having the same effect. Such a procompetitive pattern arises in our model either if the follower's cross-ownership stake increases, or if common ownership levels increase provided the leader's cross-ownership is not too high.

Second, by softening follower output and entry decisions, internalization also increases the leader's incentive to deter. Relative to a benchmark where firms maximize profits independently, such a shift in leader strategy raises total output and welfare. We verify that such a procompetitive shift occurs with an increase in symmetric internalization

reflecting higher common ownership.

We extend the model to account for product differentiation and establish that the above-mentioned effects are strongest when products are more substitutable. If firms compete in prices instead of quantities, we show internalization can lead to a more efficient deterrence outcome that does not arise if goods are homogeneous. This result is noteworthy because some of the most widely discussed evidence of anticompetitive common ownership concerns situations that are plausibly described by price competition with differentiation, like airline competition on city pairs ([Azar et al. 2018](#)).

While the effects we identify arise under specific conditions, their very existence challenges the conventional wisdom that overlapping ownership is uniformly anticompetitive. These effects may be especially relevant for industries characterized by complex ownership structures, like the automotive sector, where firms move sequentially in some submarkets and capacities play an important role. Our framework also complements recent empirical evidence on entry under overlapping ownership suggesting that overlapping ownership reduces generic entry in pharmaceutical markets. While R&D is central to pharmaceutical innovation, around patent expiration other dimensions like capacity and marketing commitments play a critical role in shaping entry dynamics in line with our model. For example incumbents often launch reformulations to preempt substitution toward generics ([Branstetter et al. 2016](#)). Our analysis supports existing empirical findings regarding potentially harmful effects of drug company stakes in generic manufacturers, but also highlights how reciprocal ownership stakes may eventually mediate them.

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A Proofs

A.1 Stackelberg equilibrium

The follower's best response in (5) does not admit a closed form, so we take another approach to derive the equilibrium output expressions (8) and (9) efficiently. First we characterize the follower's inclusive best reply (ibr), $\tilde{q}_F(Q, \lambda_F)$, which does have a closed form. Then we rewrite the leader's decision as a choice of total output, which is the equilibrium outcome of interest. In addition we allow for a more general demand specification to get a more robust version of the main result highlighting the underlying economics.

Let $P(Q)$ denote an arbitrary inverse demand function, assumed to be decreasing and twice-differentiable over a range of outputs $[0, \bar{Q}]$, with curvature $\delta(Q) = \frac{QP''(Q)}{P'(Q)} > -2$. These assumptions nest the demand specification in the text for which $\delta(Q) = \delta > -1$. Let $\bar{q}(\lambda_F)$ denote the output level such that $P(Q) + \lambda_F P'(Q)Q = C$.

Then,

Lemma A.1. *For $\lambda_F < 1$ and a leader output $q_L < \bar{q}(\lambda_F)$ for which the follower is active, the ibr is*

$$\tilde{q}_F(Q, \lambda_F) = \frac{1}{1 - \lambda_F} \frac{P(Q) - C}{-P'(Q)} - \frac{\lambda_F}{1 - \lambda_F} Q, \text{ all } Q \in [q^M, \bar{q}(\lambda_F)). \quad (41)$$

Its slope

$$\frac{\partial \tilde{q}_F}{\partial Q}(Q, \lambda_F) = -\frac{1}{1 - \lambda_F} \left(\delta(Q) \frac{P(Q) - C}{-P'(Q)Q} + 1 + \lambda_F \right) \quad (42)$$

satisfies the inequality $\frac{\partial \tilde{q}_F}{\partial Q}(Q, \lambda_F) < \frac{\tilde{q}_F(Q, \lambda_F)}{Q} - 1$ and is negative if demand is strictly logconcave, i.e. if $\delta(Q) > -1$.

Total output in a Stackelberg equilibrium with an active follower is then defined im-

PLICITLY by

$$(P(Q^S) - C) \left(1 - (1 - \lambda_L) \frac{\partial \tilde{q}_F(Q^S, \lambda_F)}{\partial Q} \right) + P'(Q^S) (Q^S - (1 - \lambda_L) \tilde{q}_F(Q^S, \lambda_F)) = 0. \quad (43)$$

Proof. The restriction $\delta(Q) > -2$ implies the follower's objective is strictly concave. With the general demand specification, differentiating (4) gives

$$P(q_F^* + q_L) - C + P'(q_F^* + q_L)(q_F^* + \lambda_F q_L) = 0 \quad (44)$$

which defines the follower's best response for all $q_L < \bar{Q}$. Substituting $Q = q_F^* + q_L$ and rearranging gives the follower's inclusive best-response (41). Differentiating with respect to Q and rearranging gives the slope (42). The follower's first-order condition implies that the markup elasticity term satisfies $\frac{P(Q) - C}{-P'(Q)Q} = \lambda_F + (1 - \lambda_F) \frac{\tilde{q}_F(Q)}{Q} \leq 1$, which yields the upper bound in the proposition $\frac{\tilde{q}_F(Q, \lambda_F)}{Q} - 1$ given that $\delta(Q) > -2$ and implies a negative slope if $\delta(Q) > -1$.

The leader's problem, expressed as a total output choice, is

$$\max_{Q \in [q^M, \bar{Q}]} (P(Q) - C) (Q - (1 - \lambda_L) \tilde{q}_F(Q, \lambda_F)), \quad (45)$$

then differentiating gives (43). □

With the specification in the text (41) yields $\tilde{q}_F(Q, \lambda_F) = \frac{(\tilde{A} - (1 + \lambda_F(1 + \delta))Q^{(1 + \delta)})Q^{-\delta}}{(1 + \delta)(1 - \lambda_F)}$, and differentiating gives $\frac{\partial \tilde{q}_F}{\partial Q}(Q, \lambda_F) = \frac{-\delta \tilde{A} Q^{-(1 + \delta)} - (1 + \lambda_F(1 + \delta))}{(1 + \delta)(1 - \lambda_F)}$. Substituting these expressions into the marginal profit expression (43) gives the following condition for total equilibrium

output with an active follower

$$\begin{aligned} & \left(\tilde{A} - Q^{1+\delta} \right) \left(1 + \frac{(1 - \lambda_L) \left(\delta \tilde{A} Q^{-(1+\delta)} + (1 + \lambda_F (1 + \delta)) \right)}{(1 + \delta) (1 - \lambda_F)} \right) \\ & - (1 + \delta) Q^\delta \left(Q - \frac{(1 - \lambda_L) \left(\tilde{A} Q^{-(1+\delta)} - (1 + \lambda_F (1 + \delta)) \right) Q}{(1 + \delta) (1 - \lambda_F)} \right) = 0. \end{aligned} \quad (46)$$

If $\delta \neq 0$, the left-hand side is a quadratic in $X := \tilde{A} Q^{-(1+\delta)}$,

$$\begin{aligned} & \delta (1 - \lambda_L) X^2 + (2 (1 - \lambda_L) + (1 + \delta) (1 - \lambda_F \lambda_L)) X \\ & - (2 + \delta) ((1 - \lambda_L) + (1 + \delta) (1 - \lambda_F \lambda_L)) = 0. \end{aligned} \quad (47)$$

The lower root is negative so marginal profit crosses the horizontal axis once from above over $\mathbb{R}_+$. The unique admissible solution is therefore the lower of $\bar{q}(\lambda_F)$ and the upper root, $\tilde{A} Q^{-(1+\delta)} = \frac{-2(1-\lambda_L)+(1+\delta)(-(1-\lambda_F\lambda_L)+\sqrt{\Delta})}{2\delta(1-\lambda_L)}$ where $\Delta = 4(1 - \lambda_L)^2 + (1 - \lambda_F \lambda_L)^2 + 4(1 + \delta)(1 - \lambda_L)(1 - \lambda_F \lambda_L)$, which yields (8). If $\delta = 0$, then (46) reduces to

$$\frac{\tilde{A}(3 - \lambda_L(2 + \lambda_F)) - Q(4 - 2\lambda_L(1 + \lambda_F))}{1 - \lambda_F} = 0 \quad (48)$$

and (9) similarly follows.

With the general demand specification, the effect of internalization on total output in Stackelberg equilibrium has the following characterization.

Proposition A.2. *In a Stackelberg equilibrium with overlapping ownership and inverse demand $P(Q)$*

(i) total output, consumer surplus, and welfare are (strictly) increasing in follower internalization if the follower is active (and $\lambda_L \in (0, 1)$) and strictly decreasing in follower

internalization if the follower is inactive;

(ii) total output, consumer surplus, and welfare are (strictly) decreasing in leader internalization (if the follower is active).

Proof. For (i), in an interior equilibrium where the follower is active, $\frac{dQ^S}{d\lambda_F}$ has the sign of the partial of marginal leader profit (43) with respect to λ_F ,

$$-(1 - \lambda_L)P'(Q^S) \left(\frac{P(Q^S) - C}{P'(Q^S)} \frac{\partial^2 \tilde{q}_F}{\partial Q \partial \lambda_F}(Q^S, \lambda_F) + \frac{\partial \tilde{q}_F}{\partial \lambda_F}(Q^S, \lambda_F) \right), \quad (49)$$

where

$$\frac{\partial \tilde{q}_F}{\partial \lambda_F}(Q^S, \lambda_F) = \frac{1}{(1 - \lambda_F)^2} \left(\frac{P(Q^S) - C}{-P'(Q^S)} - Q^S \right) = -\frac{Q^S - \tilde{q}_F(Q^S, \lambda_F)}{1 - \lambda_F} \quad (50)$$

and

$$\frac{\partial^2 \tilde{q}_F}{\partial Q \partial \lambda_F}(Q^S, \lambda_F) = \frac{1}{(1 - \lambda_F)^2} \left(\delta(Q^S) \frac{P(Q^S) - C}{P'(Q^S) Q^S} - 2 \right) = \frac{1}{1 - \lambda_F} \left(\frac{\partial \tilde{q}_F}{\partial Q}(Q^S, \lambda_F) - 1 \right). \quad (51)$$

The sign depends on the bracketed term in (49). To sign it, first substitute $\frac{P(Q^S) - C}{P'(Q^S)} = -\frac{Q^S - (1 - \lambda_L)\tilde{q}_F(Q^S, \lambda_F)}{1 - (1 - \lambda_L)\frac{\partial \tilde{q}_F}{\partial Q}(Q^S, \lambda_F)}$ from the leader's first-order condition (43), noting that the denominator is necessarily positive in equilibrium. Multiplying by this denominator, (49) is positive if

$$-(Q^S - (1 - \lambda_L)\tilde{q}_F(Q^S, \lambda_F)) \frac{\partial^2 \tilde{q}_F}{\partial Q \partial \lambda_F}(Q^S, \lambda_F) + \left(1 - (1 - \lambda_L) \frac{\partial \tilde{q}_F}{\partial Q}(Q^S, \lambda_F) \right) \frac{\partial \tilde{q}_F}{\partial \lambda_F}(Q^S, \lambda_F) > 0. \quad (52)$$

Substituting $\frac{\partial \tilde{q}_F}{\partial \lambda_F}(Q^S, \lambda_F) = -\frac{Q^S - \tilde{q}_F(Q^S, \lambda_F)}{1 - \lambda_F}$ and $\frac{\partial^2 \tilde{q}_F}{\partial Q \partial \lambda_F}(Q^S, \lambda_F) = \frac{1}{1 - \lambda_F} \left(\frac{\partial \tilde{q}_F}{\partial Q}(Q^S, \lambda_F) - 1 \right)$,

simplifying, and regrouping $\frac{\partial \tilde{q}_F}{\partial Q}(Q^S, \lambda_F)$ terms, (52) holds if

$$\lambda_L \left(\tilde{q}_F(Q^S, \lambda_F) - Q^S \frac{\partial \tilde{q}_F}{\partial Q}(Q^S, \lambda_F) \right) > 0, \quad (53)$$

which follows from the ibr slope bound $\frac{\partial \tilde{q}_F}{\partial Q}(Q, \lambda_F) < \frac{\tilde{q}_F(Q, \lambda_F)}{Q} - 1$ in Lemma A.1. If the follower is inactive finally, then the other derivative $\bar{q}'(\lambda_F) < 0$ is straightforward.

For (ii), $\frac{dQ^S}{d\lambda_L}$ has the sign of the partial of marginal leader profit (43) with respect to λ_L ,

$$(P(Q^S) - C) \frac{\partial \tilde{q}_F}{\partial Q}(Q^S, \lambda_F) + P'(Q^S) \tilde{q}_F(Q^S, \lambda_F). \quad (54)$$

This derivative has the same sign as

$$P'(Q^S) \tilde{q}_F(Q^S, \lambda_F) \left(\frac{P(Q^S) - C}{-P'(Q^S) Q^S} \frac{-\frac{\partial \tilde{q}_F}{\partial Q}(Q^S, \lambda_F)}{\tilde{q}_F(Q^S, \lambda_F)/Q^S} + 1 \right) \quad (55)$$

In the bracketed expression above the mark-up elasticity term $\frac{P(Q^S) - C}{-P'(Q^S) Q^S}$ is smaller than one, and the ibr slope bound implies that the other term satisfies $\frac{-\frac{\partial \tilde{q}_F}{\partial Q}(Q^S, \lambda_F)}{\tilde{q}_F(Q^S, \lambda_F)/Q^S} \geq -1$. We conclude that the derivative is negative so the result holds, and for the inactive follower range $\bar{q}(\lambda_F)$ is independent of λ_L . $\square$