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# Waiting after the lower trigger: A Cournot–Bertrand real option note

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## Abstract

We study a perpetual entry problem in which an entrant chooses both when to enter and whether to enter through price or quantity competition against an incumbent that has already committed to quantity. The two entry modes are mutually exclusive and differ in sunk cost and discounted operating-profit coefficient. We focus on the case in which price entry is cheaper, while quantity entry is more profitable. Although price entry may then have the lower stand-alone trigger, crossing that trigger need not justify immediate entry. We provide a sufficient condition under which there exists a nonempty interval above the lower stand-alone trigger on which immediate entry in either mode is suboptimal. The entrant waits because delay preserves the option of entering later through the more profitable quantity mode. Thus, in a Cournot–Bertrand environment with uncertainty and irreversibility, the choice of strategic variable is part of the real-options timing problem itself.

**Keywords:** real options; endogenous mode choice; Cournot–Bertrand competition; entry timing; uncertainty; strategic commitment.

**JEL Classification:** C41; D25; D81; L13

# 1. Introduction

The standard perpetual real-options model implies a simple exercise rule: invest once the state variable reaches a critical threshold (Dixit and Pindyck, 1994). This rule becomes less straightforward when entry can occur through alternative and mutually exclusive modes. Exercising one mode today destroys not only the option to wait, but also the option to enter later through a different mode. In such an environment, the first stand-alone trigger to be crossed need not determine the optimal entry policy.

This note studies this issue in a Cournot–Bertrand entry problem. An incumbent has already committed to quantity, while a potential entrant chooses both when to enter and how to enter: through price competition or through quantity competition. The two entry modes differ in sunk cost and in the discounted operating-profit coefficient they generate after entry. We focus on the case in which price entry is cheaper, while quantity entry is more profitable. Price entry may therefore have the lower stand-alone trigger. Nevertheless, immediate price entry may be suboptimal, because it eliminates the option of waiting and entering later through the more profitable quantity mode.

The point is not that the entrant waits because entry is currently unprofitable. Rather, the entrant may wait even after the lower stand-alone trigger has been crossed. The reason is that the relevant comparison is not between immediate price entry and no entry; it is between immediate entry and the value of preserving flexibility across mutually exclusive strategic variables. Thus, the choice between price and quantity is not merely a post-entry strategic detail. Under uncertainty and irreversibility, it becomes part of the timing problem itself.

The paper contributes to the Cournot–Bertrand literature by adding an irreversible-entry dimension to the choice of strategic variable. Tremblay and Tremblay (2011) analyze the role of product differentiation in Cournot–Bertrand competition, while Tremblay *et al.* (2013) endogenize timing and strategic choice. We do not revisit equilibrium selection in deterministic Cournot–Bertrand games. Instead, we ask how entry timing is altered when the entrant's available price and quantity modes are interpreted as mutually exclusive real options under uncertainty. A simple Cournot–Bertrand microfoundation, provided in Appendix A, shows how the case studied in the main text can arise from differentiated-duopoly primitives.

The paper is also related to the real-options literature on mutually exclusive investment opportunities. Décamps *et al.* (2006) show that, when investment opportunities are mutually exclusive, optimal exercise need not be characterized by a single monotone trigger. We apply this logic to strategic-variable choice in a Cournot–Bertrand setting. The resulting implication is simple but economically relevant: the lower stand-alone entry threshold may be misleading because exercising the low-cost mode destroys the option to switch later to the high-return mode.

The contribution is intentionally narrow. We do not characterize the full stopping region. Instead, we provide a sufficient condition under which there exists a nonempty interval above the lower stand-alone trigger on which immediate entry in either mode is suboptimal. A numerical example illustrates the condition. The result shows that, in a Cournot–Bertrand environment with irreversible entry, timing and strategic-variable choice must be solved jointly.

The rest of the note is organized as follows. Section 2 presents the model. Section 3 derives the main result and provides a numerical example. Section 4 interprets the result in Cournot–Bertrand terms. Section 5 concludes. Appendix A provides a simple Cournot–Bertrand microfoundation for the reduced-form profit coefficients.

## 2. Setup

Let the market-opportunity state $X_t$ follow a geometric Brownian motion,

$$dX_t = \mu X_t\, dt + \sigma X_t\, dW_t,$$

with $r > \mu$, where $r$ is the discount rate. If entry occurs at time $\tau$, the entrant chooses one of two mutually exclusive modes, $P$ or $Q$. Mode $P$ represents price entry and mode $Q$ quantity entry.

If entry occurs when $X_\tau = x$, the value from immediate exercise in mode $i \in \{P, Q\}$ is

$$\Pi_i(x) = A_i x - I_i,$$

where $A_i > 0$ is the discounted operating-profit coefficient associated with mode $i$, and $I_i > 0$ is the sunk cost of entry. In the Cournot–Bertrand interpretation, $A_P$ and $A_Q$ summarize the entrant's discounted operating position under the two post-entry regimes, taking as given the incumbent's prior quantity commitment.

The reduced-form specification is useful because the real-options mechanism does not depend on a particular demand system. Nevertheless, the ranking $A_P < A_Q$, $I_P < I_Q$ has a natural Cournot–Bertrand interpretation. Price entry may require a lower setup cost, while quantity entry may involve capacity commitment, production planning, or other sunk expenditures. At the same time, when the incumbent is committed to quantity as its strategic variable, quantity entry may generate a larger operating-profit coefficient than price entry. Appendix A provides a simple differentiated-duopoly example consistent with this ranking.

If mode $i$ were the only available opportunity, the entrant would face the standard perpetual stopping problem. Let $\beta > 1$ denote the positive root of

$$\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0.$$

The stand-alone value of mode $i$ is

$$V_i(x) = \begin{cases} B_i x^{\beta}, & x < x_i^*, \\ A_i x - I_i, & x \geq x_i^*, \end{cases}$$

$$x_i^* = \frac{\beta}{\beta - 1}\frac{I_i}{A_i},$$

$$B_i = \frac{A_i x_i^* - I_i}{(x_i^*)^\beta}.$$

Hence each mode, viewed in isolation, obeys the usual single-trigger rule. In the full problem, however, the entrant chooses both the stopping time and the mode at the stopping time. The value function is

$$V(x) = \sup_\tau \mathbb{E}_x[e^{-r\tau}\max\{A_P X_\tau - I_P, A_Q X_\tau - I_Q, 0\}].$$

Relative to the one-option problem, the maximum operator reflects the entrant's ability to choose the entry mode at the stopping time, while the zero payoff captures the option not to enter immediately. Waiting therefore preserves flexibility over both timing and mode choice.

## 3. Waiting above the lower stand-alone trigger

We focus on the case

$$A_P < A_Q \text{ and } I_P < I_Q.$$

Thus, price entry is the low-cost, low-profit mode, whereas quantity entry is the high-cost, high-profit mode. These inequalities alone do not determine which stand-alone trigger is lower, because the trigger ranking depends on the ratios $I_i/A_i$. We therefore impose

$$x_P^* < x_Q^*,$$

or equivalently,

$$\frac{I_P}{A_P} < \frac{I_Q}{A_Q}.$$

Under this condition, price entry has the lower stand-alone trigger. The next proposition shows that this is still not sufficient to justify immediate entry in the full problem.

**Proposition 1.** *Assume $A_P < A_Q$, $I_P < I_Q$, and $x_P^* < x_Q^*$. If $A_Q x_Q^* - I_Q > A_P x_Q^* - I_P$, then there exists*

$$\varepsilon \in (0, x_Q^* - x_P^*)$$

*such that, for every*

$$x \in (x_Q^* - \varepsilon, x_Q^*),$$

*immediate entry is suboptimal, even though the lower stand-alone trigger $x_P^*$ has already been crossed.*

The sufficient condition requires that, at the stand-alone quantity trigger, the payoff from entering through the more profitable quantity mode already exceeds the payoff from immediate price entry. Hence, even though price entry becomes viable first, preserving access to quantity entry can make immediate price entry unattractive.

**Proof of Proposition 1.** *Consider the feasible policy under which the entrant waits until the state reaches* $x_Q^*$ *and then enters in mode* $Q$*. For any* $x < x_Q^*$*, the value of that policy is the stand-alone continuation value of mode* $Q$*,*

$$F_Q(x) = \left(\frac{x}{x_Q^*}\right)^{\beta} (A_Q x_Q^* - I_Q).$$

*At* $x = x_Q^*$*,*

$$F_Q(x_Q^*) = A_Q x_Q^* - I_Q.$$

*By assumption,*

$$A_Q x_Q^* - I_Q > A_P x_Q^* - I_P.$$

*Hence,*

$$F_Q(x_Q^*) > A_P x_Q^* - I_P.$$

*Since both* $F_Q(x)$ *and* $A_P x - I_P$ *are continuous in* $x$*, there exists* $\eta > 0$ *such that*

$$F_Q(x) > A_P x - I_P$$

*for all*

$$x \in (x_Q^* - \eta, x_Q^*).$$

*Moreover, for every* $x < x_Q^*$*, the stand-alone option value of waiting for mode* $Q$*strictly exceeds the immediate exercise payoff from mode* $Q$*:*

$$F_Q(x) > A_Q x - I_Q.$$

*Also,* $F_Q(x) > 0$ *for all* $x < x_Q^*$*. Choose*

$$\varepsilon < min\{\eta, x_Q^* - x_P^*\}.$$

*Then, for every*

$$x \in (x_Q^* - \varepsilon, x_Q^*),$$

*w*

*e*

$$F_Q(x) > max\{A_P x - I_P,\ A_Q x - I_Q,\ 0\}.$$

*h*

*Because waiting until* $x_Q^*$ *and then entering in mode* $Q$ *is feasible in the full problem,*

*a*

*the full value function satisfies*

*v*

$$V(x) \geq F_Q(x).$$

*e*

*Therefore,*

$$V(x) > max\{A_P x - I_P,\ A_Q x - I_Q,\ 0\}.$$

$>$

*Thus, immediate entry in either mode is suboptimal for all*

$*$

*,*

$$x \in (x_Q^* - \varepsilon, x_Q^*),$$

*even though the lower stand-alone trigger $x_P^*$ has already been crossed.*

■

*Proposition 1* states that crossing the lower stand-alone trigger is not sufficient for immediate entry to be optimal. When the option to enter later through the higher-return quantity mode is sufficiently valuable, the entrant may optimally wait even though price entry is already individually viable.

The following example illustrates that the sufficient condition in Proposition 1 is nonempty and can hold under simple parameter values.

## Example 1

Let $r = 0.10, \mu = 0.03, \sigma = 0.20$. Then the positive root of

$$\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0$$

is $\beta = 2$. Suppose that $A_P = 1$, $I_P = 1$, and $A_Q = 2$, $I_Q = 3$. Thus, price entry is cheaper, $I_P < I_Q$, but quantity entry has the larger operating-profit coefficient, $A_P < A_Q$.

The stand-alone triggers are

$$x_P^* = \frac{\beta}{\beta - 1}\frac{I_P}{A_P} = 2$$

and

$$x_Q^* = \frac{\beta}{\beta - 1}\frac{I_Q}{A_Q} = 3.$$

Hence, price entry has the lower stand-alone trigger. A stand-alone analysis would therefore suggest that entry becomes optimal once $x$ reaches $x_P^* = 2$.

However, at the quantity trigger,

$$A_Q x_Q^* - I_Q = 2 \cdot 3 - 3 = 3,$$

whereas

$$A_P x_Q^* - I_P = 1 \cdot 3 - 1 = 2.$$

Therefore,

$$A_Q x_Q^* - I_Q > A_P x_Q^* - I_P,$$

so the sufficient condition in Proposition 1 is satisfied.

For example, at $x = 2.5$, the value of waiting until $x_Q^* = 3$ and then entering in mode $Q$ is

$$F_Q(2.5) = \left(\frac{2.5}{3}\right)^2 (2 \cdot 3 - 3) = \frac{25}{12} \approx 2.083.$$

By contrast, the immediate payoff from price entry is

$$A_P x - I_P = 2.5 - 1 = 1.5,$$

and the immediate payoff from quantity entry is

$$A_Q x - I_Q = 2 \cdot 2.5 - 3 = 2.$$

Thus,

$$F_Q(2.5) > \max\{A_P x - I_P,\ A_Q x - I_Q,\ 0\}.$$

Although the lower stand-alone trigger $x_P^* = 2$ has already been crossed, immediate entry is not optimal. The entrant prefers to wait because doing so preserves the option of entering later through the more profitable quantity mode.

The intuition is simple. Immediate entry in the cheap price mode secures a current payoff, but it destroys the option to wait for the more profitable quantity mode. Near $x_Q^*$, preserving access to mode $Q$ can be more valuable than entering immediately in mode $P$. At the same time, because $x < x_Q^*$, immediate entry in mode $Q$ is also dominated by waiting. Hence the lower stand-alone trigger is not, in general, the correct exercise rule in the full problem.

## 4. Interpretation for Cournot–Bertrand entry

The Cournot–Bertrand implication is that the strategic-variable choice affects not only post-entry competition but also the timing of entry. A stand-alone comparison treats the price and quantity modes separately. Since price entry has the lower sunk cost and, under $x_P^* < x_Q^*$, the lower stand-alone trigger, such a comparison suggests that the entrant should enter through price once $x_P^*$ is reached.

*Proposition 1* shows why this reasoning can fail. Once the two modes are mutually exclusive, immediate price entry eliminates the possibility of entering later through quantity. Therefore, the relevant comparison is not between immediate price entry and no entry. It is between immediate entry and the value of preserving flexibility over the entrant's future strategic variable.

This distinction matters especially when quantity entry has the larger operating-profit coefficient. In that case, the entrant may prefer to wait even though price entry is already individually viable. The delay is not caused by the absence of a profitable entry mode. It is caused by the opportunity cost of locking into the lower-return strategic variable.

The result therefore adds a real-options qualification to the Cournot–Bertrand logic. In deterministic models, the choice between price and quantity determines the nature of post-entry competition. Under uncertainty and irreversibility, the same choice also changes the value of waiting. Strategic-variable choice and entry timing must therefore be solved jointly.

## 5. Concluding remarks

This note has studied a Cournot–Bertrand entry problem in which a potential entrant chooses both when to enter and whether to enter through price or quantity competition against a quantity-committed incumbent. The two entry modes are mutually exclusive and differ in sunk cost and discounted operating-profit coefficient. When price entry is cheaper but quantity entry is more profitable, price entry may have the lower stand-alone trigger. However, this lower trigger need not determine the optimal entry policy.

We have shown that, under a simple sufficient condition, there exists a nonempty interval above the lower stand-alone trigger on which immediate entry in either mode is suboptimal. The entrant waits not because entry is currently unprofitable, but because immediate entry destroys the option of switching later to the higher-return quantity mode. Thus, the value of flexibility can delay entry even after a stand-alone entry threshold has been crossed.

The broader implication is that, under uncertainty and irreversibility, the Cournot–Bertrand choice between price and quantity cannot be treated merely as a post-entry strategic-variable decision. It is part of the real-options timing problem itself. Future work could usefully characterize the full stopping region or embed the mechanism in richer models of endogenous strategic-variable choice and entry competition.

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## Appendix. A simple Cournot–Bertrand microfoundation

This appendix provides a simple example in which quantity entry yields a larger operating-profit coefficient than price entry, while price entry may involve the lower sunk cost.

Consider a differentiated duopoly with products $i = 1,2$, where firm 1 is the incumbent and firm 2 is the entrant. Let $d \in (0,1)$ measure product substitutability. The normalized inverse demand system is

$$p_i = X(a_i - q_i - dq_j), i, j = 1,2, i \neq j,$$

and

$$\theta_i \equiv a_i - c_i > 0.$$

The state variable $X$ scales operating profits. Hence, once the entrant has entered, flow profits are proportional to $X$.

Suppose first that the entrant enters in quantity. Both firms then choose quantities. The entrant's normalized operating profit is

$$\pi_2^Q / X = (\theta_2 - q_2 - dq_1)q_2.$$

The first-order conditions are

$$\theta_1 - 2q_1 - dq_2 = 0,$$

and

$$\theta_2 - 2q_2 - dq_1 = 0.$$

Solving gives the entrant's equilibrium quantity

$$q_2^Q = \frac{2\theta_2 - d\theta_1}{4 - d^2}.$$

Using the first-order condition, the entrant's normalized operating-profit coefficient under quantity entry is

$$\lambda_Q = (q_2^Q)^2 = \left(\frac{2\theta_2 - d\theta_1}{4 - d^2}\right)^2.$$

Now suppose that the entrant enters in price, while the incumbent remains committed to quantity as its strategic variable. Let $s_2 = p_2/X$ denote the entrant's normalized price. From the inverse demand system,

$$q_2 = a_2 - dq_1 - s_2.$$

The entrant's normalized operating profit is

$$\pi_2^P / X = (s_2 - c_2)(a_2 - dq_1 - s_2).$$

$c_i$. Define

$$m_2 = s_2 - c_2.$$

$$q_2 = \theta_2 - dq_1 - m_2,$$

and the entrant's first-order condition with respect to $m_2$ gives

$$m_2 = \frac{\theta_2 - dq_1}{2}.$$

Thus, in the price-entry case,

$$q_2^P = m_2 = \frac{\theta_2 - dq_1}{2}.$$

The incumbent's normalized profit is

$$\pi_1^P / X = (\theta_1 - q_1 - dq_2)q_1.$$

Substituting $q_2 = (\theta_2 - dq_1)/2$, the incumbent chooses $q_1$to maximize

$$\left(\theta_1 - \frac{d\theta_2}{2} - \left(1 - \frac{d^2}{2}\right) q_1\right) q_1.$$

The incumbent's equilibrium quantity is therefore

$$q_1^P = \frac{2\theta_1 - d\theta_2}{4 - 2d^2}.$$

Hence the entrant's equilibrium quantity under price entry is

$$q_2^P = \frac{(4 - d^2)\theta_2 - 2d\theta_1}{4(2 - d^2)}.$$

The entrant's normalized operating-profit coefficient under price entry is

$$\lambda_P = (q_2^P)^2 = \left(\frac{(4 - d^2)\theta_2 - 2d\theta_1}{4(2 - d^2)}\right)^2.$$

To see that quantity entry may yield the larger operating-profit coefficient, consider the symmetric case

$$\theta_1 = \theta_2 = \theta > 0.$$

Then

$$q_2^Q = \frac{\theta}{2 + d},$$

whereas

$$q_2^P = \frac{\theta(4 - 2d - d^2)}{4(2 - d^2)}.$$

∈

)

$$q_2^Q - q_2^P = \frac{\theta d^3}{4(2 + d)(2 - d^2)} > 0.$$

Therefore,

$$q_2^Q > q_2^P,$$

and hence

$$\lambda_Q > \lambda_P.$$

If post-entry flow profits are $\lambda_i X_t$, $i = P, Q$, and $r > \mu$, the discounted operating-profit coefficient is

$$A_i = \frac{\lambda_i}{r - \mu}.$$

Thus,

$$\lambda_Q > \lambda_P$$

implies

$$A_Q > A_P.$$

The sunk-cost ranking is independent of

$$I_P < I_Q$$

because price entry may require less capacity commitment or production infrastructure than quantity entry. Hence the reduced-form case studied in the main text,

$$A_P < A_Q, I_P < I_Q,$$

can arise from standard Cournot–Bertrand primitives.