



ISSN 1791-3144

DEPARTMENT OF ECONOMICS

DISCUSSION PAPER SERIES

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Discussion Paper No. 02/2023

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A Composite Indicator of Social Inclusion for EU based on the Inverted BoD Model

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ABSTRACT: In this paper, after reviewing the existing modeling approaches for reverse indicators, we opt for the use of the inverted Benefit-of-the-doubt (IBoD) model when all the considered indicators are reverse, as is the case with composite indicators related to social inclusion. Using EU data for 2014 we provide comparative empirical results using the IBoD model and three data transformations employed by previous studies. We also provide a thorough analysis of the performance of EU countries during the period 2011-2020 in terms of the composite indicator obtained from the IBoD model. Our empirical results indicate that the relative performance across EU has neither improved nor deteriorated and there is evidence supporting local instead of global convergence in social inclusion with certain groups of countries.

KEYWORDS: Social Inclusion; EU, Reverse Indicators; Inverted BoD Model

May 2023

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1. Introduction

During the last three decades, social inclusion has been considered as an important priority for achieving further development within the European Union (EU). Its importance is more highlighted in turbulent times, such as the global financial and debt crisis, the Covid-19 pandemic, and the recent hikes in energy prices. Formally defined by the European Commission (2004, p. 10) as the process of ensuring that citizens at risk of poverty and exclusion are able to enjoy (i) full participation in economic, social and cultural life, (ii) an adequate standard of living and well-being, and (iii) greater participation in decision making, social inclusion had received a central role in European social policy as early as the Amsterdam Treaty of 1997 (Atkinson *et al.*, 2004). This public interest was concretely formalized in the Lisbon Strategy --launched in 2000, in which a scheme for policy co-ordination in promoting social inclusion was proposed as part of a larger plan to harmonize European Social policies. In particular, a set of commonly agreed and defined indicators related to social inclusion was first considered and the Open Method of Coordination (OMC) was adopted to monitor the performance of EU countries.¹ This involved the establishment of National Action Plans, which are submitted biannually and outlined a country's proposed actions for reducing poverty and fostering social inclusion over the next two years (Rogge and Konttinen, 2018). Ever since, promoting social inclusion has been at the top of every major EU agenda, such as in the 'Europe 2020' program (the successor of the Lisbon Strategy), in which a commitment for lifting at least 20 million people in the EU out of risk of poverty or social exclusion until 2020 was undertaken (see European Commission 2013), and the 'European Pillar of Social Rights', through which EU countries pledged to three headline targets concerning

education, employment and poverty reduction up to 2030 (see European Commission, 2021).² This has spawned the interest of the academic community in exploring various features of social inclusion in terms of both measurement and policy analysis, such as the challenges faced by European countries in implementing such policies, establishing quantitative targets, and monitoring changes in the performance of countries (see Rogge, 2017 and the references therein). Of significant help in this direction is the harmonized database of indicators monitoring dimensions of social inclusion for the EU countries, maintained by Eurostat³

It is generally agreed that tools measuring social inclusion should be able to adequately capture its complex and multidimensional nature (Atkinson *et al.*, 2002). The performance of a country across the various dimensions of social inclusion had been shown to differ a lot (see, e.g., Marlier *et al.*, 2012; Giambona and Vassallo, 2014). As such, comparable effort was devoted to the development of a composite indicator that could provide an overall picture of the multiple dimensions of social inclusion. The Benefit-of-the-Doubt (BoD) model, a special form of the non-parametric Data Envelopment Analysis (DEA), was mainly used for such purpose (see Giambona and Vassallo, 2014; Rogge, 2017; van Puyenbroeck and Rogge, 2017; Rogge and Konttinen, 2018; and Rogge and Self, 2019), as its main features (country-specific, best-possible aggregation weights, no input dimension) resonate well with the voluntary nature of the OMC and respect the countries' diversity in setting priorities and choosing policy measures. However, the Eurostat social inclusion database includes almost exclusively deprivation indicators that provide estimates of *social exclusion* instead of inclusion. These are known in the literature as *reverse* indicators in the sense of Lewis and Sexton (2004), i.e., those for which larger values indicate worse rather than better performance. Such indicators cannot be included *ad hoc* in the BoD model, which naturally accommodates *forward* indicators, i.e., those for which larger values indicate better performance.⁴ To deal with this, previous studies evaluating social inclusion in the EU have used different data transformations before the relevant variables are included in the BoD model. These include (i) a variant of the min-max transformation (Giambona and Vassallo, 2014), (ii) subtracting the values of the reverse indicators from 100 (Rogge, 2017; Rogge and Konttinen, 2018; Rogge and Self, 2019), and (iii) using the inverse values of the reverse indicators (van Puyenbroeck and Rogge, 2017).

Notice however that social inclusion is not the only case where the considered component indicators are reverse indicators. Other cases include the construction of composite indicators for water quality (Whittaker *et al.*, 2015), traffic fatality (Bastos *et al.*, 2015), greenhouse gases (Edmonds *et al.*, 2017), well-being (Herero-Prieto *et al.*, 2020), and vulnerability of climate change (Edmonds *et al.*, 2020) and regions (Fernades-Macho *et al.*, 2020; Blancard *et al.*, 2021). In contrast to studies on social inclusion, where the data for the reverse indicators are transformed before they are used in the BoD model, the above studies treated reverse indicators as either bad outputs or inputs in several variants of pure inputs or outputs DEA models.

The objective of this paper is threefold: *first*, we critically review all previous approaches used to deal with reverse indicators, not restricted to the case of social inclusion. These can be classified into two broad categories, one related to the different treatment of the reverse indicators in pure inputs or outputs DEA models, e.g., Whittaker *et al.* (2015), Bastos *et al.* (2015) and Edmonds *et al.* (2017, 2020), and another involving the transformation of the data related to reverse indicators, such as those we referred to for the construction of the social inclusion composite indicator. Several interesting results arise from this task: (i) the models used by Whittaker *et al.* (2015), Bastos *et al.* (2015) and Edmonds *et al.* (2017, 2020) are in fact equivalent to each other and they may be referred to as the inverted BoD (IBoD) model; (ii) the BoD model with only reverse component indicators rates of Decision-Making Units (DMUs) as infinitely inefficient; and (iii) in the case of only reverse indicators, the values of the composite indicator obtained by the IBoD model will always be larger than or equal to those obtained by the BoD model using the inverse values of the reverse indicators.

Second, we provide comparative empirical results for the EU social inclusion composite indicator using the three aforementioned data transformations employed in previous studies, i.e., a variant of the min-max transformation, subtracting the values of the reverse indicators from 100, and using the inverse values of the reverse indicators, and the IBoD model. Our empirical findings confirm the theoretical results concerning the IBoD model and the BoD model using the inverse values of the reverse indicators and also provide several interesting insights about the relation between the IBoD model and the BoD model using either the min-max transformation or that subtracting the values of the reverse indicators from 100.

Third, we analyze the performance of 27 EU countries during the period 2011-2020 in terms of the social inclusion composite indicator obtained from the IBoD model. Our empirical results indicate that, despite individual “success stories”, the relative performance across EU has neither improved nor deteriorated during the previous decade. On the contrary, we find evidence supporting local instead of global convergence in social inclusion performance with certain groups of countries.

The rest of the paper is organized as follows: In the next section, we critically review the approaches used previously for modeling reverse indicators. Our empirical results regarding the EU social inclusion composite indicator are discussed in the third section, while the last section concludes.

2. Methods and Materials

In this section we firstly present the BoD model and then we discuss and compare the alternative ways of treating reverse indicators.

2.1 The BoD model

The BoD model (see Cherchye *et al.*, 2007), which is commonly used to aggregate forward composite indicators, may be considered as an input-oriented, constant-returns-to-scale (CRS) model with a single constant input (set without loss of generality to be equal to one for all DMUs), the multiplier and the envelopment forms of which are given as (see e.g. Karagiannis, 2021):

$$\begin{aligned}
 \max_{u_i^k} \quad & \sum_{i=1}^L v_i^k I_i^k \\
 \text{st} \quad & \sum_{i=1}^L v_i^k I_i^h \leq 1 \quad h = 1, \dots, K \\
 & v_i^k \geq 0 \quad i = 1, \dots, L
 \end{aligned}
 \qquad
 \begin{aligned}
 \min_{\mu_i^k} \quad & \sum_{h=1}^K \mu_h^k \\
 \text{st} \quad & \sum_{h=1}^K \mu_h^k I_i^h \geq I_i^k \quad i = 1, \dots, L \\
 & \mu_h^k \geq 0 \quad h = 1, \dots, K
 \end{aligned}
 \tag{1}$$

where I refers to indicators, v to indicators' multipliers, μ to intensity variables, $i = 1, \dots, L$ is used to index indicators, and $h = 1, \dots, k, \dots, K$ to index DMUs. The BoD model results, for each DMU, in a set of positive aggregation weights that maximize the weighted average of component indicators to reflect their forward nature and when they are used by any other DMU result in composite indicator that is less than or equal to one. Hence, the greater the value of the composite indicator is the better

the achievements of the evaluated DMU, with the value of one indicating the best possible performance.

2.2 Treatment of reverse indicators

In several occasions, some or even all of the considered component indicators to be aggregated into a composite indicator reflect negative performance aspects and for this reason, they may be referred to as reverse or non-isotonic indicators. Following Lewis and Sexton (2004), for reverse indicators, larger values are associated with worse achievements or unfortunate events. In contrast, for forward or isotonic indicators, which reflect positive performance, larger values are associated with better achievements. This distinct feature of reverse indicators requires a different modeling strategy compared to forward indicators.

In the literature, two alternative approaches have been used to treat reverse indicators: the first treats them as either reverse (bad) outputs or forward inputs within different variants of the pure inputs or outputs DEA models and the second transforms the relevant data, so that larger values indicate better achievements, and includes the transformed variables in the conventional BoD model. In this section, we review these approaches when all component indicators are reverse, as in the case of social inclusion composite indicator.

In the first approach, three variants of the pure inputs or outputs DEA models have been used, in all of which reverse indicators are treated as (forward) inputs. First, Whittaker *et al.* (2015) favored the input-oriented, CRS, single constant output (set without loss of generality to be equal to one for all DMUs) model of Thompson *et al.* (1986), which in its multiplier and envelopment form is given as:

$$\begin{aligned}
 \max_{v_i^k, u_0^k} \quad & u^k \\
 \text{st} \quad & \sum_{i=1}^I v_i^k I_i^k = 1 \\
 & \sum_{i=1}^I v_i^k I_i^h - u^k \geq 0 \quad h = 1, \dots, K \\
 & v_i^k \geq 0 \quad i = 1, \dots, L \\
 & u^k \geq 0
 \end{aligned}
 \qquad
 \begin{aligned}
 \min_{\lambda_j^k, \phi^k} \quad & \phi^k \\
 \text{st} \quad & \sum_{h=1}^K \mu_h^k I_i^h \leq \phi^k I_i^k \quad i = 1, \dots, L \\
 & \sum_{h=1}^K \mu_h^k 1^h \geq 1^k \\
 & \mu_h^k \geq 0 \quad h = 1, \dots, K
 \end{aligned}
 \tag{2}$$

where u refer to unit vector's multipliers and ϕ to input-oriented technical efficiency scores. *Second*, Bastos *et al.* (2015) used the output-oriented, CRS, single constant output model of Doyle *et al.* (1995), which in its multiplier and envelopment form is given as:

$$\begin{aligned}
\min_{v_i^k, u^k} \quad & \sum_{i=1}^I v_i^k I_i^k & \max_{\lambda_j^k, \theta^k} \quad & \theta^k \\
st \quad & u^k = 1 & st \quad & \sum_{h=1}^K \lambda_h^k 1^h \geq 1^k \theta^k \\
& \sum_{i=1}^I v_i^k I_i^h - u^k \geq 0 \quad h = 1, \dots, K & & \sum_{h=1}^K \lambda_h^k I_i^h \leq I_i^k \quad i = 1, \dots, L \\
& v_i^k \geq 0 \quad i = 1, \dots, L & & \lambda_h^k \geq 0 \quad h = 1, \dots, K \\
& u^k \geq 0 & &
\end{aligned} \tag{3}$$

where θ refers to output-oriented technical efficiency scores and λ to intensity variables. *Third*, Edmonds *et al.* (2017, 2020) employed the input-oriented, variable-returns-to-scale (VRS), pure inputs DEA model of Adolphson *et al.* (1991), which in its multiplier and envelopment form is given as:

$$\begin{aligned}
\max_{v_i^k, u_0^k} \quad & -u_0^k & \min_{\lambda_j^k, \phi^k} \quad & \phi^k \\
st \quad & \sum_{i=1}^I v_i^k I_i^k = 1 & st \quad & \sum_{h=1}^K \mu_h^k I_i^h \leq \phi^k I_i^k \quad i = 1, \dots, L \\
& \sum_{i=1}^I v_i^k I_i^h + u_0^k \geq 0 \quad h = 1, \dots, K & & \sum_{h=1}^K \mu_h^k = 1 \\
& v_i^k \geq 0 \quad i = 1, \dots, L & & \mu_h^k \geq 0 \quad h = 1, \dots, K \\
& u_0^k \text{ free in sign} & &
\end{aligned} \tag{4}$$

where u_0 is a parameter related to returns to scale.

The aforementioned three models are related to each other. In particular, Karagiannis (2021) has showed that the input-oriented VRS model without outputs in (4), the input-oriented CRS model with a single constant output in (2), and the inverse of the output-oriented CRS model with a single constant output in (3) are all equivalent to each other. This means that the Whittaker *et al.* (2015), Bastos *et al.* (2015), and Edmonds *et al.* (2017, 2020) models will result in the same estimates of the underlying composite indicator. We may thus say that this strand of the literature

opts for what Färe and Karagiannis (2014) referred to as the inverted BoD (IBoD) model.

Based on (3) and after few manipulations, we re-state the multiplier and the envelopment forms of the IBoD model as:

$$\begin{aligned}
 \min_{v_i^k} \quad & \sum_{i=1}^L v_i^k I_i^k \\
 \text{st} \quad & \sum_{i=1}^L v_i^k I_i^h \geq 1 \quad h = 1, \dots, K \\
 & v_i^k \geq 0 \quad i = 1, \dots, L
 \end{aligned}
 \qquad
 \begin{aligned}
 \max_{\lambda_j^k} \quad & \sum_{h=1}^K \lambda_h^k \\
 \text{st} \quad & \sum_{h=1}^K \lambda_h^k I_i^h \leq I_i^k \quad i = 1, \dots, L \\
 & \lambda_h^k \geq 0 \quad h = 1, \dots, K
 \end{aligned}
 \tag{5}$$

The IBoD model results, for each DMU, in a set of positive aggregation weights that minimize the weighted average of component indicators to reflect their reverse nature and when they are used by any other DMU result in composite indicator that is greater than or equal to one. Hence, the lower the value of the composite indicator is the better the achievements of the evaluated DMU, with the value of one indicating the best possible performance.

On the other hand, Färe *et al.* (2019) proposed a modification of the BoD model that can include both forward and reverse component indicators and claimed (p. 395) that “... one can also treat with the proposed model cases where there are only reverse indicators ...”. The multiplier and the envelopment forms of the Färe *et al.* (2019) model are given as:

$$\begin{aligned}
 \max_{u_i^k} \quad & \sum_{i=1}^l u_i^k I_i^k - \sum_{i=l+1}^L u_i^k I_i^k \\
 \text{st} \quad & \sum_{i=1}^l u_i^k I_i^k - \sum_{i=l+1}^L u_i^k I_i^k \leq 1^k \quad h = 1, \dots, K \\
 & u_i \geq 0 \quad i = 1, \dots, L
 \end{aligned}
 \qquad
 \begin{aligned}
 \min_{\lambda_k} \quad & \sum_{h=1}^K \lambda_h^k \\
 \text{st} \quad & \sum_{h=1}^K \lambda_h^k I_i^h \geq I_i^k \quad i = 1, \dots, l \\
 & \sum_{h=1}^K \lambda_h^k I_i^h \leq I_i^k \quad i = l+1, \dots, L \\
 & \lambda_h^k \geq 0 \quad h = 1, \dots, K
 \end{aligned}
 \tag{6}$$

where the considered component indicators are grouped into $1, \dots, l$ that are forward and $l+1, \dots, L$ that are reverse in nature. If we consider only reverse indicators, (6) is reduced to

$$\begin{aligned}
& \max_{u_i^k} - \sum_{i=l+1}^L u_i^k I_i^k & \min_{\lambda_k} \sum_{h=1}^K \lambda_h^k & (7) \\
& st \quad - \sum_{i=l+1}^L u_i^k I_i^k \leq 1^k \quad h = 1, \dots, K & st \quad \sum_{h=1}^K \lambda_h^k I_i^h \leq I_i^k \quad i = l+1, \dots, L \\
& u_i \geq 0 \quad i = l+1, \dots, L & \lambda_h^k \geq 0 \quad h = 1, \dots, K
\end{aligned}$$

or equivalently to

$$\begin{aligned}
& \min_{u_i^k} \sum_{i=l+1}^L u_i^k I_i^k & \min_{\lambda_k} \sum_{h=1}^K \lambda_h^k & (8) \\
& st \quad \sum_{i=l+1}^L u_i^k I_i^k \geq -1^k \quad h = 1, \dots, K & st \quad \sum_{h=1}^K \lambda_h^k I_i^h \leq I_i^k \quad i = l+1, \dots, L \\
& u_i \geq 0 \quad i = l+1, \dots, L & \lambda_h^k \geq 0 \quad h = 1, \dots, K
\end{aligned}$$

Alternatively, we can derive (7) or (8) from a Lewis and Sexton (2004) type of input-oriented CRS model with forward inputs and reverse outputs, in which all forward inputs are set equal to one for all DMUs. From (8) we may infer the following:

PROPOSITION 1: The BoD model with only reverse component indicators rates of DMUs as infinitely inefficient.

PROOF: Based on the envelopment form, we can see that (8) has a feasible solution for $\lambda_h^k = 0, h = 1, \dots, k, \dots, K$, which is also optimal, since with $\lambda_h^k \geq 0$ the lowest value of the objective function $\sum_{h=1}^K \lambda_h^k$ can be obtained if all $\lambda_h^k = 0$. Thus, (8) rates all DMUs as infinitely inefficient, i.e., as having a zero efficiency score. The same results is also obtained by using the multiplier form: as all component indicators and optimal weight have non-negative values, i.e., $I_i^k \geq 0$ and $u_i^k \geq 0$, a solution with all weights being equal to zero is both feasible and optimal, since it guarantees the lowest value (zero) for the objective function.

Thus, the BoD model is not an appropriate modeling choice when all considered component indicators are reverse because it results in a composite indicator that is equal to zero for all DMUs.

The second approach applies some kind of data transformation to the reverse component indicators before they are used in the conventional form of the BoD

model, i.e., (1). In particular, three forms of transformation have been used so far: *first*, the max-min normalization $(\max_k I_i^k - I_i^k)/(\max_k I_i^k - \min_k I_i^k)$, (e.g., Giambona and Vassallo, 2014); *second*, $100 - I_i^k$ (e.g., Rogge, 2017; Rogge and Kottinen, 2018; Rogge and Slef, 2019); and *third*, $1/I_i^k$ (e.g., van Puyenbroeck and Rogge, 2017). This way, it is argued, reverse indicators are “transformed” into forward ones and as a result, the BoD model can be used for obtaining the underlying composite indicator.

2.3 Comparison of the two approaches

In this sub-section, we compare the IBoD model in (5) with the conventional BoD model in (1) after substituting into it the aforementioned three forms of data transformation. By substituting either of the first two of these data transformations, which are additive, into the envelopment form of the BoD model, we have:

$$\begin{aligned} \min_{\mu_i^k} \quad & \sum_{h=1}^K \mu_h^k \\ \text{st} \quad & \sum_{h=1}^K \mu_h^k I_i^h \leq I_i^k + A \left(\sum_{h=1}^K \mu_h^k - 1 \right) \quad i = 1, \dots, L \\ & \mu_h^k \geq 0 \quad h = 1, \dots, K \end{aligned} \quad (9)$$

where A is equal to $\max_k I_i^k$ in the case of the max-min normalization while is equal to 100 for the second transformation. Comparing (9) with (5) we gain almost no insights about the values of resulting composite indicators as these two models have different objective functions as well as different constraints.

Consider, on the other hand, the BoD model in (1) when the inverse values of the reverse indicators are used instead of original values, i.e.:

$$\begin{aligned} \max_{u_i^k} \quad & \sum_{i=1}^L u_i^k \left(\frac{1}{I_i^k} \right) \\ \text{st} \quad & \sum_{i=1}^L u_i^k \left(\frac{1}{I_i^k} \right) \leq 1 \quad h = 1, \dots, K \\ & u_i^k \geq 0 \quad i = 1, \dots, L \end{aligned} \quad \begin{aligned} \min_{\mu_i^k} \quad & \sum_{h=1}^K \mu_h^k \\ \text{st} \quad & \sum_{h=1}^K \mu_h^k \left(\frac{1}{I_i^h} \right) \geq \left(\frac{1}{I_i^k} \right) \quad i = 1, \dots, L \\ & \mu_h^k \geq 0 \quad h = 1, \dots, K \end{aligned} \quad (10)$$

Given the equivalence between the BoD model and the output-oriented, VRS, single constant input model of Fernandez-Castro and Smith (1994) model (see Karagiannis, 2021), (10) is equivalent to

$$\begin{aligned}
& \min_{u_i^k, v^k, v_0^k} v^k - v_0^k & \max_{\lambda_j^k, \theta^k} \theta^k & (11) \\
& st \quad \sum_{i=1}^L u_i^k \left(\frac{1}{I_i^k} \right) = 1 & st \quad \sum_{h=1}^K \lambda_h^k \left(\frac{1}{I_i^h} \right) \geq \theta^k \left(\frac{1}{I_i^k} \right) \quad i = 1, \dots, L \\
& \quad \sum_{i=1}^L u_i^k \left(\frac{1}{I_i^h} \right) \leq v^k - v_0^k \quad h = 1, \dots, K & \quad \sum_{h=1}^K \lambda_h^k = 1 \\
& \quad u_j^k \geq 0 \quad i = 1, \dots, L & \quad \lambda_h^k \geq 0 \quad h = 1, \dots, K \\
& \quad v^k \geq 0 \\
& \quad v_0^k \text{ free in sign}
\end{aligned}$$

where v_0 is a parameter related to returns to scale. On the other hand, notice that the envelopment form of (4) may equivalently be written as:

$$\begin{aligned}
& \max_{\lambda_j^k, \hat{\phi}^k} \hat{\phi}^k & (12) \\
& st \quad \left(\sum_{h=1}^K \mu_h^k I_i^h \right)^{-1} \geq \hat{\phi}^k \left(\frac{1}{I_i^k} \right) \quad i = 1, \dots, L \\
& \quad \sum_{h=1}^K \mu_h^k = 1 \\
& \quad \mu_h^k \geq 0 \quad h = 1, \dots, K
\end{aligned}$$

where $\hat{\phi}^k = 1/\phi^k$. Now we can compare (11) with (12) as their envelopment forms differ only in the left-hand side of their inequality constraint. This results into:

PROPOSITION 2: In the case of only reverse indicators, the values of the composite indicator obtained from the IBoD model will always be larger than or equal to those obtained by the BoD model using the inverse values of the reverse indicators.

PROOF: From Jensen's inequality for convex functions, we have $(\sum_{h=1}^K \mu_h^k I_i^h)^{-1} \leq \sum_{h=1}^K \lambda_h^k (1/I_i^h)$. Consequently, the targets set by (10) are higher than those set by (11) and as a result, $\hat{\phi}^k \leq \theta^k$ for $k = 1, \dots, K$.

An immediate implication of the above proposition is that the BoD model using the inverse values of the reverse indicators tends to underestimate DMUs' performance

compared to the IBoD model. This concerns also DMUs that are identified as best performers by the IBoD model, namely those that are assigned a composite indicator value of one. By definition, for these DMUs, an outperforming convex combination of some other DMUs does not exist in the context of the IBoD model but it might or might not exist when the inverse quantities of their indicators are used in the BoD model. In fact, we can distinguish between two groups of DMUs that are rated as efficient by the IBoD model, namely those performing the best (i.e., have the lowest quantity) in one of the considered component indicators, and the remaining efficient DMUs. Then, we may show that DMUs in the former group belong to the frontier of both the IBoD model and the BoD model using the inverse values of the reverse indicators:

PROPOSITION 3: If $I_i^k = \min_h I_i^h$ for some $i = 1, \dots, L$, then the k^{th} DMU is rated as efficient by both the IBoD model and the BoD model using the inverse values of the reverse indicators.

Proof: If $I_i^k = \min_h I_i^h$, then $1/I_i^k = \max_h (1/I_i^h)$. Given that I_i^k , for $i = 1, \dots, L$, are treated as inputs in (3) and $(1/I_i^k)$, for $i = 1, \dots, L$, are treated as outputs in (10), the proof that $\varphi^k = \theta^k = 1$ follows directly from Theorem 4.3 in Cooper, Seiford and Tone (2006, p.89).

Thus, for each such DMU, a convex combination of some other of the existing DMUs that weakly outperforming it does not exist either in the IBoD model or in the BoD model using the inverse values of the reverse indicators. This however is not true for the remaining DMUs rated as efficient by the IBoD model, which means that some of them may lose their efficiency status when evaluated by the BoD model using the inverse values of the reverse indicators.

We illustrate this in Figure 1, which portrays the efficiency frontiers for the IBoD model and the BoD model using the inverse values of the reverse indicators when $i = 2$. The IBoD frontier, presented as an input-oriented CRS model with a single constant output as in (5), is given in the first quadrant in Figure 1. All DMUs are assumed to be efficient. In the second and the fourth quadrant, a unit rectangular hyperbola is used to generate the inverse quantities of the two reverse indicators. These are then used to derive the efficient frontier of the BoD model using the inverse values of the reverse indicators, which is presented as an output-oriented CRS model

with a single constant output. We can see that the two DMUs performing the best (i.e., having the lowest quantities) for the two indicators are on the frontiers generated by both models. On the other hand, the remaining three DMUs that are efficient in the first quadrant of Figure 1 drop from the frontier when in the third quadrant.

[please place Figure 1 here]

To examine this case further, we consider three simple numerical examples, in which a number of DMUs are evaluated based on two reverse component indicators. The relevant data are given in the second and third columns of Table 1, while the IBoD efficiency scores are given in the fourth column. In addition, the inverse values are given in the fifth and sixth columns of Table 1 and the efficiency scores obtained from the BoD model using the inverse values of the reverse indicators, referred to as BoD-III henceforth, are given in the last column. The relevant efficiency frontiers for the IBoD and BoD-III models are portrayed in Figure 2 (in a similar fashion to Figure 1). For ease of exposition, DMUs are depicted by using upper case letters in the first quadrant and by using lower case letters in the third quadrant. In addition, different linetypes are used to portray the efficient frontiers in each example: a solid linetype is used in the first example, a dashed in the second and a dotted in the third.

[please place Table 1 here]

From Table 1 we see that, in all three numerical examples, the BoD-III efficiency scores are lower than or equal to those obtained from the IBoD model, as suggested by Proposition 1. The two DMUs performing the best (i.e., having the lowest quantity) in each of the two indicators (DMUs A and C) are efficient in all three numerical examples either with the IBoD or the BoD-III model, as suggested by Proposition 2. In each example, one additional DMU is rated as efficient by the IBoD model. In the first example it is the DMU B1, which however is rated as inefficient in the BoD-III model. In the second example, DMU B2 is included instead of DMU B1. DMU B2 is rated as efficient by both models, giving rise to the dashed kinked frontier in the third quadrant of Figure 2. In the third example, DMUs B1 and B2 are both excluded and then DMU D is rated as efficient by the IBoD model. DMU D has quantities equal to 1.5 for both indicators and it may be viewed as linear combination of DMUs A and C. However, the inverse value of its indicators (each of which is

given as $1/I_i^D = 1/(0.5I_i^A + 0.5I_i^C)$, $i = 1, 2$, are outperformed by the average of DMUs' A and C inverse indicators, i.e., it holds that $1/I_i^D = 1/(0.5O_i^A + 0.5I_i^C) < 0.5(1/I_i^A + 1/I_i^C)$, for $i = 1, 2$. Consequently, DMU D is rated as inefficient by the BoD-III model.

[please place Figure 2 here]

3. Data and Empirical Results

3.1 Data Description

The importance placed on promoting social inclusion in the EU (through the Lisbon Strategy and more recently through the Europe 2020 Strategy) has resulted in the development of a harmonized database for monitoring Member States' progress in several social inclusion dimensions. These dimensions are almost exclusively measured through deprivation indicators, i.e., those providing estimates of *social exclusion* instead of inclusion. In Eurostat's employment and social inclusion indicators database, all indicators provided except one (life expectancy) are measures of relative deprivation.

We use data from Eurostat's employment and social inclusion indicators database on four indicators: (i) share of people at risk of poverty rate after social transfers (measured as the percentage of population having an equivalized income below 60% of the median after social transfers), (ii) material deprivation rate (measured as the percentage of population with an enforced lack of at least three out of nine material deprivation items in the 'economic strain and durables' dimension), (iii) people aged 18-59 living in jobless households (measured as the share of persons aged 18-59 who are living in households where no-one works), and (iv) early leavers from education and training (measured as the share of the population aged 18 to 24 with at most lower secondary education who were not involved in any education or training during the four weeks preceding the survey).⁵ These indicators are among the "Level 2" or "primary indicators" for social inclusion included in Eurostat's database and in essence, they correspond to the operational objectives of the European social inclusion strategy (Giambona and Vassallo, 2014). For example, three of these indicators (the share of people at risk of poverty rate after social transfers, the material deprivation rate, and the share of people aged 18-59 living in jobless households) are

used by the Commission to construct a composite indicator of at-risk-of-poverty and social exclusion (AROPE), which is the main tool for monitoring the performance of EU member countries (Rogge and Self, 2019). They have also been used by all previous studies using variants of the BoD model to assess social inclusion. In particular, Giambona and Vassallo (2014), van Puyenbroeck and Rogge (2017), and Rogge and Konttinen (2018) used these four indicators and data at the EU Member State level spanning from 2006 to 2014, Rogge (2017) used seven indicators including these four and data at the EU Member State level from 2007 to 2012, while Rogge and Self (2019) used three of these --namely those comprising the AROPE index-- and data from 176 regions including NUTS-2, NUTS-1 and NUTS-0 (country level) regions from the 27 EU Member States and Iceland, Norway, Switzerland, Turkey and the UK.

Our analysis involves 28 DMUs, namely all 27 EU Member States plus the EU27 aggregate data, and is conducted separately for each year from 2011 to 2020. The sample average values of the four Eurostat indicators for each year are given in Table 2, while more detailed descriptive statistics are given in Table A1 in the Appendix. In addition, Figure 3 compares the performance of every country across each component indicator for the years 2011 and 2020. Countries located on the right of the dashed 45° line have improved their performance in the considered indicator and appear as green-colored dots, while countries appearing as red-colored dots are located on the left of the dashed 45° line, indicating that their performance has deteriorated over time. From these we can see that improvements took place during the last decade in three out of the four component indicators. In particular, considerable reduction in the share of people with an enforced lack of material deprivation items took place (the median share dropped from about 20% in 2011 to 12.5% in 2020), which is however mostly attributed to the catching up of the poorest performing countries (see the changes in the maximum and minimum values in Table A1). In total, 22 of the 27 countries improved their performance in this component indicator from 2011 to 2020 (see Figure 3b). The shares of people living in jobless households and of early leavers from education and training decreased on average across the EU as well. Quantitatively, these improvements were less than those in the material deprivation rate and were attributed to improvements in both top- and bottom-performing countries. From Figures 3c and 3d we see that both the share of people living in jobless households and the share of early leavers from education and

training declined in 20 out of the 27 countries. On the other hand, the performance of countries in terms of reducing the share of people at risk of poverty rate has somewhat worsened over time, as indicated by the increase in the median values from 15.5% to 16.1% (see Table A1). In total, the share of people at risk of poverty rate increased -- indicating performance deterioration-- in 12 of the 27 countries (see Figure 3a).

The changes in the standard deviation provide some insights on the catching-up between poor and top performers. From Table A1 we can see that EU countries converged to each other in terms of the material deprivation rate and the share of early leavers from education and training. This can also be seen from Figures 3b and 3d, where the poorest performing countries are those with the largest improvements over time (such as Bulgaria, Romania, and Latvia for the material deprivation rate and Greece, Portugal and Spain for the share of early leavers from education and training). On the other hand, the dispersion in the shares of people living in jobless households remained stable, while divergence between country performance appears to be the case for the share of people at risk of poverty.

[please place Table 2 here]

[please place Figure 3 here]

3.2 Comparative Empirical Results

The efficiency scores obtained from the three alternative data transformations and the IBoD model, for 2014, are given in columns two to five of Table 3 and portrayed in Figure 4. In the following, the efficiency scores (i.e., the value of the composite indicator) from the BoD model using 100 minus the value of the reverse indicator are referred to as BoD-I, those from the BoD model using the min-max transformation as BoD-II, and those from the BoD model using the inverse values of the reverse indicators as BoD-III. The number in parentheses next to each score denotes the country's rank, while changes in rank between pairs of approaches are given in columns six to eleven of Table 3.

[please place Table 3 here]

[please place Figure 4 here]

We see that, on average, BoD-I and BoD-II result in higher values of the composite indicator compared to the other two models. In addition, the IBoD model results on average in higher values of the composite indicator compared to the BoD-III model, as suggested by Proposition 1. In IBoD, BoD-II and BoD-III, seven countries have a composite indicator equal to one, which means that they are best performers in terms of social inclusion services provision. Three of them (Czech Republic, Croatia, and Sweden) perform the best in (i.e., have the smallest quantities of) the four reverse indicators used. The remaining four countries (Denmark, Luxemburg, Netherlands, and Slovenia) appear to be somewhat all-around good performers.⁶ In terms of Figure 2, the first group of countries corresponds to DMUs A and C, while the second one to DMUs B1 and B2. On the contrary, four of these seven countries are also efficient in the BoD-III model. These are Czech Republic, Croatia, and Sweden, the results for which confirm Proposition 2, and Luxemburg. In terms of Figure 2, Luxemburg corresponds to DMU B2, while Denmark, the Netherlands, and Slovenia –which drop from the frontier-- to DMU B1.

The different models have unavoidably affected country rankings. As rankings usually make headlines and attract media attention, rank changes can have important ramifications for a country's reputation and subsequent policy decisions. Average shift in ranks (given as $R = (\frac{1}{N}) \sum_{j=1}^N |rank_1^j - rank_2^j|$ (Saisana *et al.*, 2005), are larger when comparing the BoD-III model with the other three models than when comparing them with one another (see Table 3). This is to some extent due to the reduction of the number of efficient countries in the BoD-III model. The countries dropping from the frontier when this model is used lose from four to seven rank positions compared to the rankings of the other three models. The ranks of inefficient countries are also affected: France, Hungary and Finland gain one to three positions in the BoD-III model compared to each of the other three models, while Lithuania loses one to four rank positions. Comparing the IBoD model with the other three, we see that Belgium, Germany, and Estonia are relatively favored by the IBoD model, while Greece, Cyprus, Latvia and Portugal gain up to two rank positions compared to all other specifications.

Regarding differences in the values of the composite indicator and focusing on the IBoD and BoD-III models, we see that the latter underestimates the performance of countries by 3.6% on average. Despite that the differences in the distribution of the

values of the composite indicator between these two models appear not to be statistically significant (see Table 4 for the Banker and Natarajan (2011) statistical tests), there exist countries exhibiting quite substantial reduction in their estimates in the BoD-III model compared to IBoD. The largest losses are found for Austria (10.5%), Slovenia (10.3%), and Poland (9.7%). Eight additional countries exhibit more than 4% reduction in their composite indicator value. It is important to notice that 9 among these 11 countries are found to be inefficient in both models. This indicates that using the BoD-III model has the potential to affect quite a lot the values of the composite indicator regardless of their status as efficient or inefficient in the IBoD model. To quantify the extent of this, let us use Austria as an example. Its IBoD-based composite indicator value is 0.940, meaning that best practices will be attained if the quantities of the four reverse indicators are reduced by $1 - 0.94 = 6\%$. The BoD-III model assigns to Austria a composite indicator value of 0.832. This means that the inverse of each indicator should be increased by 20% ($1/0.832 = 1.20$), or alternatively that each reverse indicator should be reduced by $1 - 0.832 = 16.8\%$. Thus, the country should, for example, aim at reducing the share of people living in jobless households from 8.6% down to 7.15% (8.6×0.832). This would require quite heavier investments than reducing it to 8.08% (8.6×0.940), as is indicated by the IBoD model.

[please place Table 4 here]

The alternative approaches for treating reverse indicators affect also the sets of peers for the inefficient countries (see Table 5). This has important ramifications for these countries' social inclusion policy design, which relies much on the "examples to follow". The number of countries with different sets of peers ranges from 8 (between the IBoD and BoD-I models) and 17 (between the BoD-II and BoD-III models). For three countries, each of the four models designates a different set of peers. It is interesting to notice that two among these are Spain and Italy, for which no efficient country is identified as a peer by all the four different models. On the other hand, there are six countries for which the four models identify the same set of peers, but each model assigns a different CI score to each country.

[please place Table 5 here]

3.3 EU Social inclusion over 2011-2020.

The empirical results in this section are based on the IBoD model and cover the period 2011 to 2020. Each year is treated as a separate sample for which the resulting values of the composite indicator and the rankings are given in Table 6. In that, the average estimates for groups of countries sharing a similar form of welfare state model are also provided.⁷ From Table 6 we see that, on average, the performance of countries has improved only slightly over the last decade, while the value of the composite indicator for the EU27 show that as a whole its relative performance has neither improved nor worsened over the last 10 years. Given that the average values in three out of the four reverse indicators used in the analysis have decreased during the same period (see Table 2), this indicates that the EU countries have been progressing with more or less the same pace so that gaps in relative performance among them have neither increased nor shrunk, i.e., catching up or convergence appears not to have taken place. These findings complement earlier results in the literature indicating that improvements in performance of some EU countries up to 2012 were balanced out by worsening performance in others (Rogge, 2017). They also suggest that the slight progress and convergence in terms of social inclusion documented in earlier studies for the years up to 2014 (Rogge, 2017; Rogge and Konttinen, 2018) appears to have not continued over the years to 2020.

[please place Table 6 here]

In particular, the countries forming the efficient frontier have not changed significantly over the decade: three countries (Czech Republic, Luxemburg, and Slovenia) constitute best practices throughout the whole period, while Sweden and Croatia become efficient in respectively 2012 and 2014 and they have remained in the frontier since then. These countries perform the best in terms of one or more social inclusion indicators in at least one year. In addition, Denmark and Netherlands are on the frontier for respectively three and six out of the 10 years but remain fairly close to it in the remaining years (their values of the composite indicator never drop below 0.890). These two countries are not the best in a particular indicator but constitute examples of all-around good performers in the provision of social inclusion. On the other hand, Bulgaria, Romania, Spain and Italy always appear on the bottom of the rankings. These are largely in line with the results presented in earlier studies (see,

e.g., Giambona and Vassallo, 2014; Rogge, 2017; van Puyenbroeck and Rogge, 2017), in which countries such as Czech Republic, Slovenia, Sweden, Luxemburg and the Netherlands are ranked on the top and Bulgaria and Spain are found as very poor performers.⁸

For several inefficient countries, negligible progress or regress has occurred over the period 2011-2020. This is the result of (i) humped trajectories involving an increase in relative performance followed by a decline (as in Finland and Austria) or the other way around (as in Portugal), or (ii) more volatile movements involving frequent ups and downs in performance (as in France, Germany, Estonia, Romania, and Spain). Spain's value of the composite indicator is somewhat lower in 2020 compared to 2011, while the reverse is true for Romania and Estonia. For France and Austria, the differences are negligible. Notable exceptions in this pattern are Greece and Ireland, for which marked progress took place, and Cyprus, which was on the frontier in 2011 but ranks 20th out of the 28 countries in 2020. For Cyprus, this regress is also documented elsewhere (Rogge and Konttinen, 2018). For the other two countries, progress over the 2010-2020 period appears to be in contract with the regress documented in these countries over the years prior to 2010 (Giambona and Vassallo, 2014). Other countries for which the values of the composite indicator gradually increased are Lithuania, Portugal, Hungary, Latvia, Belgium and Bulgaria. However, this progress is not reflected in their rankings (only Hungary gains two rank positions in 2020 compared to 2011). On the other hand, Germany and Italy ranked quite lower in 2011 compared to 2020. For Germany this is due to a marked deterioration of performance in 2020, while for Italy, the value of the composite indicator declined until 2018 (indicating that the distance between Italy's performance and that of the frontier countries widened) and has not ever since caught up with their 2011 levels.

A detailed examination of country group average values of the composite indicator show that local --instead of global-- convergence seems to be the case. In particular, excluding the Anglo-Saxon group that includes only Ireland, we may distinguish between three broad groups of countries: the first comprises of the Nordic countries, which outperforms on average all other groups over the period 2011-2020. Relative performance in this group improved much between 2013-2017 (reaching levels close to one) before returning to its 2011 level (above 0.910) in 2020. The second group consists of the post-communist and the Northern European countries.

These groups perform on average below the Nordic group, better than the remaining two groups, and appear to have gradually converged to each other in levels between 0.850 and 0.900. More specifically, Northern European countries performed better than post-communist ones for most years, but performance in the latter gradually catches up and even outruns the former in 2020. The third group contains the Southern European and the former USSR countries, which appear to also converge to each other, albeit to a lower level of performance compared to the other two groups. The gap between the two groups is the widest in 2012, when the former USSR group outperforms on average the Southern European countries. This resonates with earlier findings indicating a performance regress in this group up to 2012, which is most likely a result of austerity measures during the debt crisis episodes in these countries, which have put social protection systems under stress (Rogge, 2017). Then, average performance in the two groups appears to converge in a level of CI scores between 0.650 and 0.700.

4. Concluding Remarks

The aim of this paper is to analyze social inclusion in the EU during the last decade by means of a composite indicator. For doing so, we first critically review all previous approaches used to deal with reverse indicators. These include three different variants of the pure inputs or outputs DEA model as well as strategies related to the transformation of the data related to reverse indicators. From this we found that (i) the three variants of the pure inputs or outputs DEA models used by previous studies are in fact equivalent to each other and coincide with the IBoD model; (ii) the BoD model with only reverse component indicators rates of DMUs as infinitely inefficient and thus it is not an appropriate modeling choice; and (iii) in the case of only reverse indicators, the values of the composite indicator obtained by the IBoD model will always be larger than or equal to those obtained by the BoD model using the inverse values of the reverse indicators.

We also provide comparative empirical results for the EU using the IBoD model and three data transformations employed by previous studies, namely, a variant of the min-max transformation, subtracting the values of the reverse indicators from 100, and constructing the inverse values of the reverse indicators. Overall, our empirical findings confirm the theoretical results concerning the IBoD model and the BoD model using the inverse values of the reverse indicators. In addition, we found

that on average, BoD-I and BoD-II results in large efficiency scores compared to the other two, and that the average shift in ranks is larger when comparing the BoD-III model with the other three models than when comparing them with one another.

Lastly, we provide a thorough analysis of the performance of EU countries during the period 2011-2020 in terms of the social inclusion composite indicator obtained from the IBoD model. Our empirical results indicate that, despite individual “success stories”, the relative performance across EU has neither improved nor deteriorated during the previous decade. In contrast, we find evidence supporting local instead of global convergence in social inclusion performance with certain groups of countries.

The current form of the IBoD model used to monitor the performance of EU countries in terms of social inclusion is however not without limitations, as it inherits certain deficiencies of the conventional BoD and DEA models. More specifically, it is sensitive to the presence of outliers, and it does not account for the effect of background ‘contextual’ variables which are not under the direct control of countries but can create favorable operating conditions for some of them and unfavorable for others. Such variables could be, for instance, the age-dependency ratio and the share of single-parent households (see Rogge and Self, 2019). Hence, future work will extend the present paper’s results via a robust order-m framework (see Cazals *et al.* 2002) to mitigate the impact of outlying observations and through a conditional DEA framework (see Daraio and Simar 2005) to account for the effect of contextual variables.

In addition, the paper analyses performance over time but treats each year as a separate sample. As an anonymous reviewer noted, the performance of a country in terms of social inclusion in a given year is, to some extent, affected from the performance in the previous years. More appropriate approaches have been developed within the DEA framework for evaluating the intertemporal performance of countries. These include the use of the Malmquist productivity index (Caves *et al.*, 1982; Färe *et al.*, 1994), other quantity indices (see, e.g., van Puyenbroeck and Rogge, 2017) and window analysis (see Charnes *et al.*, 1984). Some of these have been used in evaluating social inclusion in the EU (see Rogge, 2017 and Rogge and Konttinen, 2018) without however treating the reverse indicators in the appropriate way (i.e., using the IBoD model). The task of appropriately evaluating the intertemporal

performance of countries in terms of social inclusion is another interesting avenue for future research.

Table 1: Data and efficiency scores for the three numerical examples

DMU	I_1	I_2	IBoD efficiency score	$1/I_1$	$1/I_2$	BoD-III efficiency score
illustrative example 1						
A	0.500	2.500	1.000	0.400	2.000	1.000
B1	1.000	1.000	1.000	1.000	1.000	0.833
C	2.500	0.500	1.000	2.000	0.400	1.000
D	1.500	1.500	0.667	0.667	0.667	0.556
E	2.000	1.250	0.696	0.800	0.500	0.542
F	2.500	2.000	0.471	0.500	0.400	0.375
illustrative example 2						
A	0.500	2.500	1.000	0.400	2.000	1.000
B2	0.800	0.800	1.000	1.250	1.250	1.000
C	2.500	0.500	1.000	2.000	0.400	1.000
D	1.500	1.500	0.533	0.667	0.667	0.533
E	2.000	1.250	0.587	0.800	0.500	0.528
F	2.500	2.000	0.386	0.500	0.400	0.363
illustrative example 3						
A	0.500	2.500	1.000	0.400	2.000	1.000
C	2.500	0.500	1.000	2.000	0.400	1.000
D	1.500	1.500	1.000	0.667	0.667	0.556
E	2.000	1.250	0.923	0.800	0.500	0.542
F	2.500	2.000	0.667	0.500	0.400	0.375

Note: The IBoD model uses I_1 and I_2 as reverse indicators while the BoD-III model uses $1/I_1$ and $1/I_2$ as forward indicators.

Table 2: Average values of model variables

year	people at risk of poverty rate	material deprivation rate	people living in jobless households	early leavers from education and training
2011	16.300	22.441	10.204	11.248
2012	16.311	23.248	10.559	10.881
2013	16.441	23.096	10.785	10.207
2014	16.870	21.700	10.507	9.689
2015	17.085	19.922	10.137	9.722
2016	17.096	18.289	9.656	9.300
2017	16.796	16.741	8.941	9.267
2018	16.722	14.922	8.307	9.037
2019	16.326	13.404	7.922	8.870
2020	16.241	13.352	8.404	8.678

Notes: a) averages are calculated over the 27 EU Member states, b) all variables are measured in percentages of the total population except “people living in jobless households”, which is measured as a percentage of people aged 18-59.

Table 3: Efficiency estimates and rank changes, alternative approaches for treating reverse social inclusion indicators, 2014.

Country	model ^a	rank changes between models								
	(1) IBoD	(2) BoD-I	(3) BoD-II	(4) BoD-III	(1)-(2)	(1)-(3)	(1)-(4)	(2)-(3)	(2)-(4)	(3)-(4)
EU27	0.654 (20)	0.949 (23)	0.709 (23)	0.598 (23)	-3	-3	-3	0	0	0
Belgium	0.768 (16)	0.968 (19)	0.858 (18)	0.719 (17)	-3	-2	-1	1	2	1
Bulgaria	0.477 (28)	0.929 (26)	0.529 (28)	0.477 (28)	2	0	0	-2	-2	0
Czechia	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	0	0	0	0	0	0
Denmark	1.000 (1)	1.000 (1)	1.000 (1)	0.964 (5)	0	0	-4	0	-4	-4
Germany	0.871 (10)	0.989 (11)	0.927 (11)	0.818 (10)	-1	-1	0	0	1	1
Estonia	0.805 (13)	0.981 (15)	0.880 (15)	0.768 (14)	-2	-2	-1	0	1	1
Ireland	0.730 (19)	0.974 (18)	0.855 (19)	0.672 (19)	1	0	0	-1	-1	0
Greece	0.541 (27)	0.942 (25)	0.675 (25)	0.498 (26)	2	2	1	0	-1	-1
Spain	0.547 (26)	0.913 (28)	0.684 (24)	0.496 (27)	-2	2	-1	4	1	-3
France	0.857 (12)	0.981 (14)	0.913 (12)	0.809 (11)	-2	0	1	2	3	1
Croatia	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	0	0	0	0	0	0
Italy	0.556 (25)	0.922 (27)	0.573 (27)	0.526 (25)	-2	-2	0	0	2	2
Cyprus	0.759 (18)	0.979 (17)	0.874 (16)	0.720 (16)	1	2	2	1	1	0
Latvia	0.649 (21)	0.967 (20)	0.797 (20)	0.649 (20)	1	1	1	0	0	0
Lithuania	0.764 (17)	0.981 (16)	0.884 (14)	0.680 (18)	1	3	-1	2	-2	-4
Luxembourg	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	0	0	0	0	0	0
Hungary	0.647 (22)	0.962 (22)	0.717 (22)	0.647 (21)	0	0	1	0	1	1
Malta	0.794 (15)	0.982 (13)	0.871 (17)	0.786 (13)	2	-2	2	-4	0	4
Netherlands	1.000 (1)	1.000 (1)	1.000 (1)	0.957 (6)	0	0	-5	0	-5	-5
Austria	0.940 (9)	0.993 (9)	0.968 (9)	0.832 (9)	0	0	0	0	0	0
Poland	0.859 (11)	0.990 (10)	0.939 (10)	0.762 (15)	1	1	-4	0	-5	-5
Portugal	0.637 (23)	0.963 (21)	0.724 (21)	0.636 (22)	2	2	1	0	-1	-1
Romania	0.560 (24)	0.949 (24)	0.622 (26)	0.560 (24)	0	-2	0	-2	0	2
Slovenia	1.000 (1)	1.000 (1)	1.000 (1)	0.897 (8)	0	0	-7	0	-7	-7
Slovakia	0.803 (14)	0.984 (12)	0.903 (13)	0.787 (12)	2	1	2	-1	0	1
Finland	0.958 (8)	0.995 (8)	0.983 (8)	0.919 (7)	0	0	1	0	1	1
Sweden	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	0	0	0	0	0	0
average ^b	0.792	0.975	0.853	0.756	1.071	1.000	1.393	0.714	1.464	1.607

Note: a) The IBoD model treats the indicators as reverse. The other three models transform the reverse indicators into forward using the min-max transformation (BoD-I), subtracting indicator values from 100 (BoD-II), and taking their inverse (BoD-III). b) average rank change is calculated over the absolute values of rank change, see Saisana *et al.* (2005).

Table 4: Statistical tests, alternative approaches for treating reverse social inclusion indicators, 2014

treatment of reverse indicators				
	(i) as inputs	(ii) 100-minus	(iii) min-max	(iv) inverse
(i)		9.933***	1.476*	1.183
(ii)	87.054***		6.727***	11.752***
(iii)	1.847*	47.138***		1.747**
(iv)	1.238	107.796***	2.287**	

Notes: (a) F1 (F2) test compares the distributions of efficiency scores with each other assuming an exponential (half-normal) distribution of the efficiency scores (see Banker and Natarajan, 2011). (b) Results from the F1 (F2) test are depicted in the upper (lower) diagonal. (c) Three, two and one stars denote statistical significance at 1%, 5% and 10% respectively

Table 5: Efficient peers, alternative approaches for treating reverse social inclusion indicators, 2014

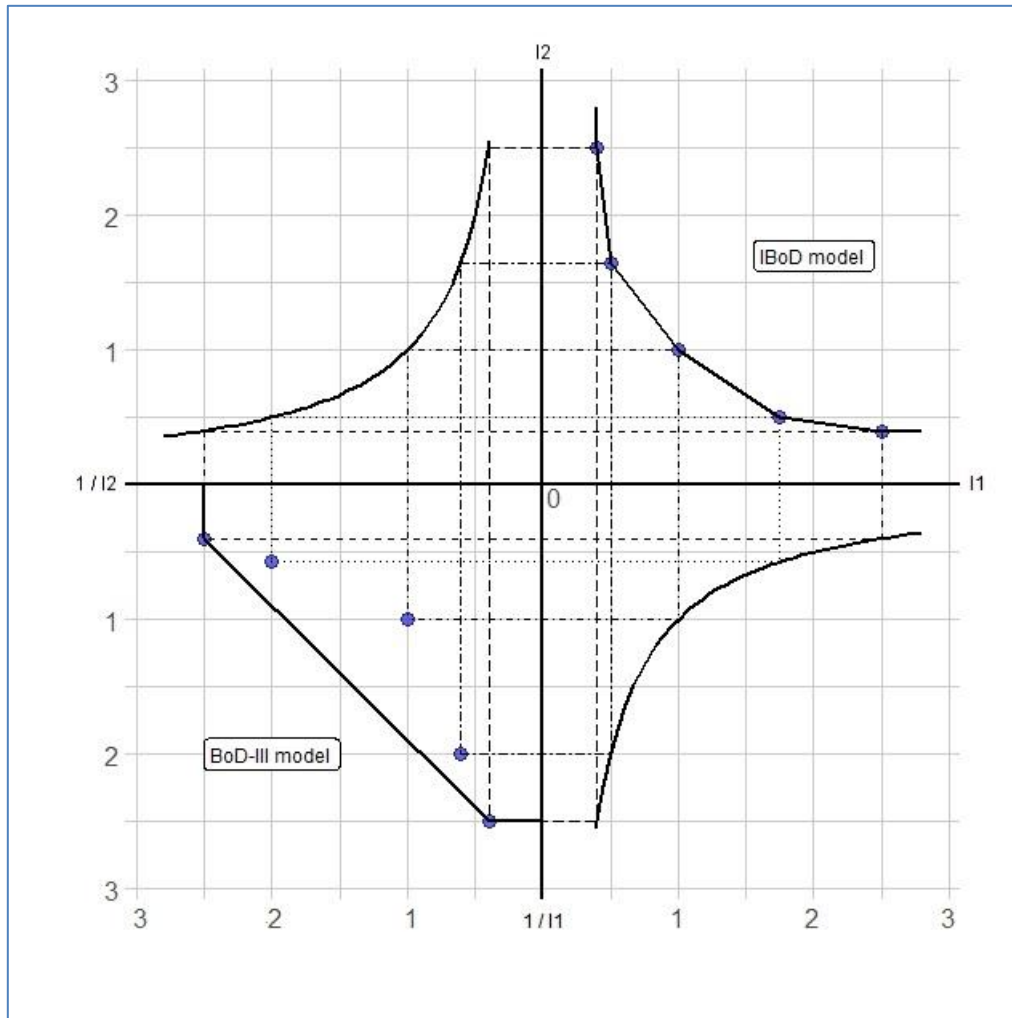
country	model			
	IBoD	BoD-I	BoD-II	BoD-III
(1) EU27	(4), (17), (20)	(4), (17)	(4), (5), (17), (20)	(4), (28)
(2) Belgium	(4), (5), (28)	(4), (5), (28)	(5), (28)	(4), (28)
(3) Bulgaria	(4)	(4)	(4), (12)	(4)
(4) Czechia	(4)	(4)	(4)	(4)
(5) Denmark	(5)	(5)	(5)	(4), (28)
(6) Germany	(4), (17)	(4), (17)	(4), (17)	(4), (17)
(7) Estonia	(4), (17)	(4), (17)	(4), (17)	(4), (17)
(8) Ireland	(4), (12), (25)	(4), (12), (25)	(12), (25)	(4), (12), (28)
(9) Greece	(4), (12)	(4), (12)	(12)	(4), (12)
(10) Spain	(4), (17), (20)	(4), (17)	(28)	(4), (28)
(11) France	(4), (5), (20)	(4), (5), (28)	(4), (5), (28)	(4), (28)
(12) Croatia	(12)	(12)	(12)	(12)
(13) Italy	(4), (17), (20)	(4)	(5), (17), (28)	(4), (28)
(14) Cyprus	(4), (12)	(4), (12)	(4), (12)	(4), (12)
(15) Latvia	(4)	(4), (12)	(4), (12)	(4)
(16) Lithuania	(4), (12), (25)	(4), (12)	(4), (12), (25)	(4), (12), (17)
(17) Luxembourg	(17)	(17)	(17)	(17)
(18) Hungary	(4)	(4)	(4)	(4)
(19) Malta	(4), (17)	(4)	(4), (17)	(4), (17)
(20) Netherlands	(20)	(20)	(20)	(4), (28)
(21) Austria	(4), (5), (17), (28)	(4), (5), (17), (28)	(4), (5), (17), (28)	(4), (12), (17), (28)
(22) Poland	(4), (12), (25)	(4), (12), (25)	(4), (12), (25)	(4), (12), (17)
(23) Portugal	(4), (17)	(4)	(4)	(4), (17)
(24) Romania	(4)	(4)	(4)	(4)
(25) Slovenia	(25)	(25)	(25)	(4), (12), (28)
(26) Slovakia	(4), (12)	(4), (12)	(4), (12)	(4), (12)
(27) Finland	(5) (17), (28)	(5) (17), (28)	(5), (28)	(4), (28)
(28) Sweden	(28)	(28)	(28)	(28)

Note: The IBoD model treats the indicators as reverse. The other three models transform the reverse indicators into forward using the min-max transformation (BoD-I), subtracting indicator values from 100 (BoD-II), and taking their inverse (BoD-III).

Table 6: Social inclusion estimates using the IBoD model, 2011-2020

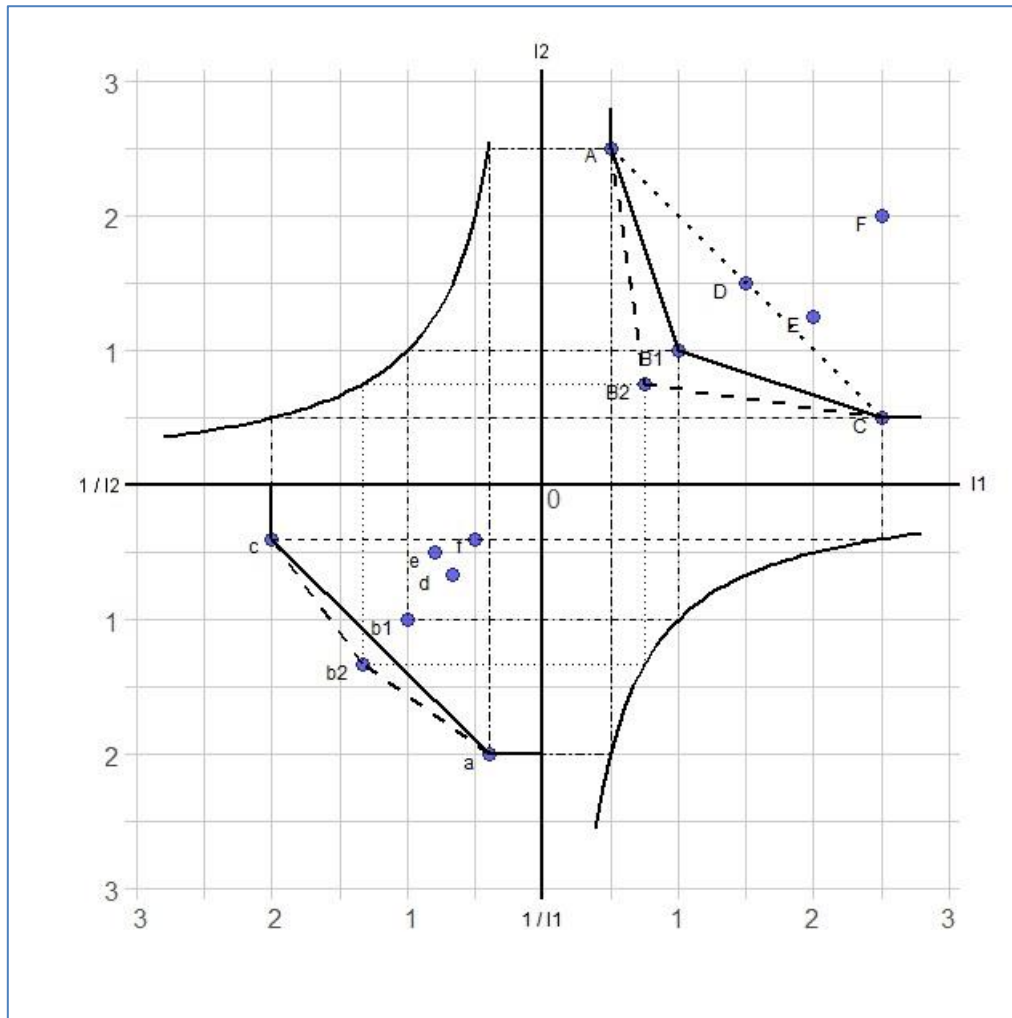
country	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
EU27	0.640 (21)	0.623 (21)	0.617 (20)	0.654 (20)	0.632 (23)	0.623 (22)	0.624 (23)	0.585 (24)	0.628 (24)	0.634 (23)
Nordic countries	0.913	0.932	0.970	0.986	0.990	0.990	0.990	0.945	0.977	0.912
Denmark	0.973 (6)	0.909 (8)	0.970 (6)	1.000 (1)	0.986 (7)	1.000 (1)	1.000 (1)	0.932 (8)	0.985 (6)	0.892 (11)
Finland	0.836 (12)	0.887 (9)	0.940 (7)	0.958 (8)	0.983 (8)	0.970 (10)	0.969 (9)	0.901 (10)	0.945 (9)	0.846 (14)
Sweden	0.929 (7)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)
Northern-European countries	0.859	0.851	0.885	0.906	0.905	0.888	0.875	0.856	0.884	0.843
Belgium	0.699 (17)	0.678 (18)	0.742 (15)	0.768 (16)	0.769 (19)	0.748 (18)	0.739 (17)	0.698 (21)	0.753 (19)	0.759 (18)
Germany	0.813 (13)	0.805 (14)	0.873 (9)	0.871 (10)	0.852 (13)	0.866 (12)	0.804 (13)	0.814 (15)	0.859 (14)	0.646 (22)
France	0.760 (15)	0.707 (15)	0.799 (12)	0.857 (12)	0.832 (15)	0.805 (17)	0.786 (14)	0.716 (18)	0.769 (18)	0.758 (19)
Luxembourg	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)
Netherlands	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	0.986 (7)	0.991 (7)	0.962 (6)	0.952 (8)	1.000 (1)
Austria	0.880 (10)	0.913 (7)	0.898 (8)	0.940 (9)	0.978 (9)	0.922 (11)	0.933 (10)	0.948 (7)	0.972 (7)	0.895 (9)
Southern-European countries	0.716	0.648	0.596	0.639	0.678	0.684	0.666	0.670	0.683	0.696
Greece	0.478 (28)	0.475 (27)	0.475 (28)	0.541 (27)	0.628 (24)	0.715 (20)	0.695 (19)	0.829 (14)	0.885 (12)	0.852 (13)
Spain	0.570 (24)	0.488 (26)	0.515 (26)	0.547 (26)	0.540 (26)	0.553 (25)	0.570 (25)	0.512 (25)	0.532 (25)	0.461 (28)
Italy	0.587 (23)	0.553 (25)	0.524 (25)	0.556 (25)	0.518 (27)	0.501 (26)	0.499 (26)	0.473 (27)	0.502 (28)	0.517 (27)
Cyprus	1.000 (1)	0.867 (10)	0.674 (18)	0.759 (18)	0.891 (11)	0.746 (19)	0.681 (20)	0.706 (19)	0.702 (20)	0.664 (20)
Malta	0.907 (8)	0.855 (12)	0.805 (11)	0.794 (15)	0.849 (14)	0.979 (8)	0.971 (8)	0.850 (12)	0.797 (17)	0.896 (8)
Portugal	0.754 (16)	0.650 (20)	0.582 (22)	0.637 (23)	0.644 (22)	0.610 (24)	0.580 (24)	0.650 (22)	0.679 (22)	0.787 (15)
Anglo-Saxon countries	0.655	0.606	0.616	0.730	0.783	0.825	0.857	0.875	0.912	0.880
Ireland	0.655 (20)	0.606 (23)	0.616 (21)	0.730 (19)	0.783 (18)	0.825 (16)	0.857 (12)	0.875 (11)	0.912 (10)	0.880 (12)
Post-communist countries	0.815	0.819	0.788	0.827	0.852	0.855	0.828	0.848	0.868	0.888
Bulgaria	0.484 (27)	0.471 (28)	0.477 (27)	0.477 (28)	0.472 (28)	0.462 (28)	0.489 (27)	0.470 (28)	0.514 (27)	0.561 (26)
Czechia	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)
Croatia	0.840 (11)	0.863 (11)	0.867 (10)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)
Hungary	0.695 (18)	0.671 (19)	0.573 (23)	0.647 (22)	0.699 (20)	0.704 (21)	0.679 (21)	0.750 (17)	0.821 (16)	0.772 (16)
Poland	0.793 (14)	0.810 (13)	0.798 (13)	0.859 (11)	0.939 (10)	0.970 (9)	0.891 (11)	0.927 (9)	0.894 (11)	0.991 (7)
Slovenia	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)
Slovakia	0.895 (9)	0.918 (6)	0.798 (14)	0.803 (14)	0.858 (12)	0.849 (13)	0.734 (18)	0.787 (16)	0.849 (15)	0.893 (10)
Former USSR countries	0.595	0.650	0.666	0.694	0.712	0.695	0.645	0.656	0.692	0.653
Estonia	0.677 (19)	0.691 (17)	0.738 (16)	0.805 (13)	0.826 (16)	0.843 (15)	0.747 (15)	0.701 (20)	0.675 (23)	0.759 (17)
Latvia	0.561 (25)	0.619 (22)	0.660 (19)	0.649 (21)	0.667 (21)	0.621 (23)	0.640 (22)	0.615 (23)	0.693 (21)	0.621 (24)
Lithuania	0.613 (22)	0.699 (16)	0.704 (17)	0.764 (17)	0.809 (17)	0.846 (14)	0.745 (16)	0.832 (13)	0.874 (13)	0.656 (21)
Romania	0.531 (26)	0.591 (24)	0.564 (24)	0.560 (24)	0.547 (25)	0.472 (27)	0.449 (28)	0.476 (26)	0.528 (26)	0.573 (25)
Average (EU-27 Member States)	0.775	0.768	0.763	0.797	0.817	0.815	0.794	0.794	0.818	0.803

Figure 1: The efficiency frontiers obtained from the IBoD and BoD-III models.



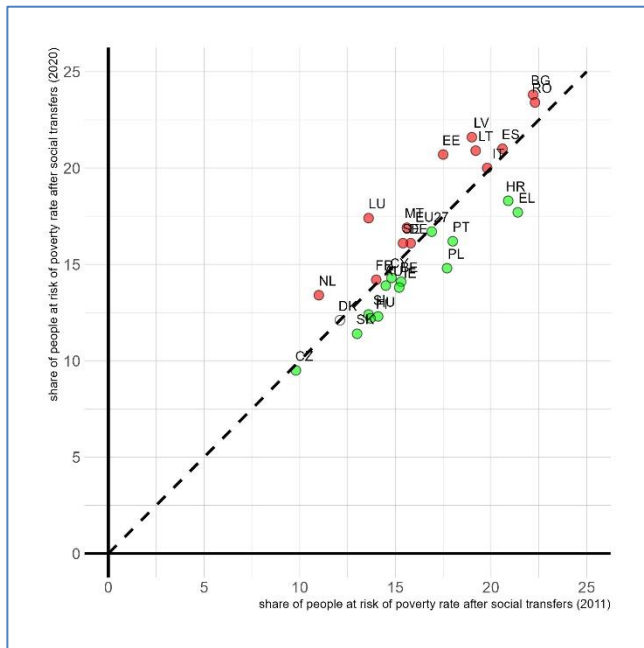
Note: The IBoD model uses I_1 and I_2 as reverse indicators while the BoD-III model uses $1/I_1$ and $1/I_2$ as forward indicators

Figure 2: The efficiency frontiers obtained from the IBoD and BoD-III models for the three numerical examples.

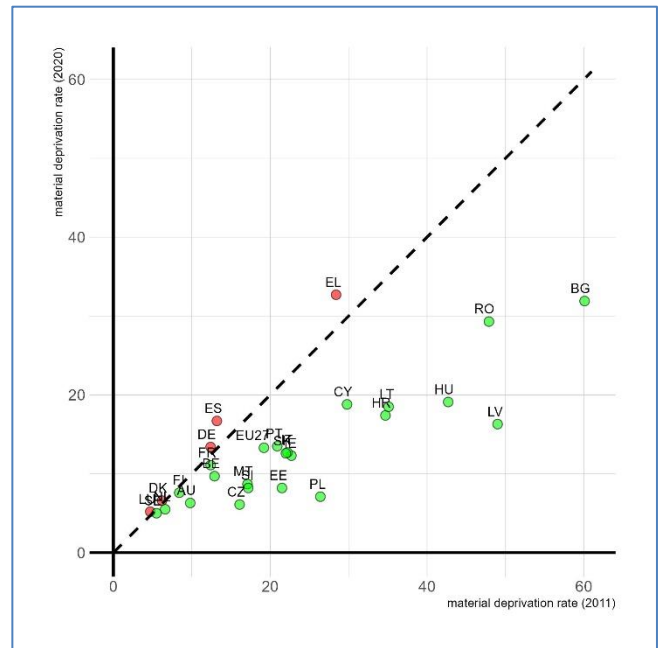


Note: The IBoD model uses I_1 and I_2 as reverse indicators while the BoD-III model uses $1/I_1$ and $1/I_2$ as forward indicators.

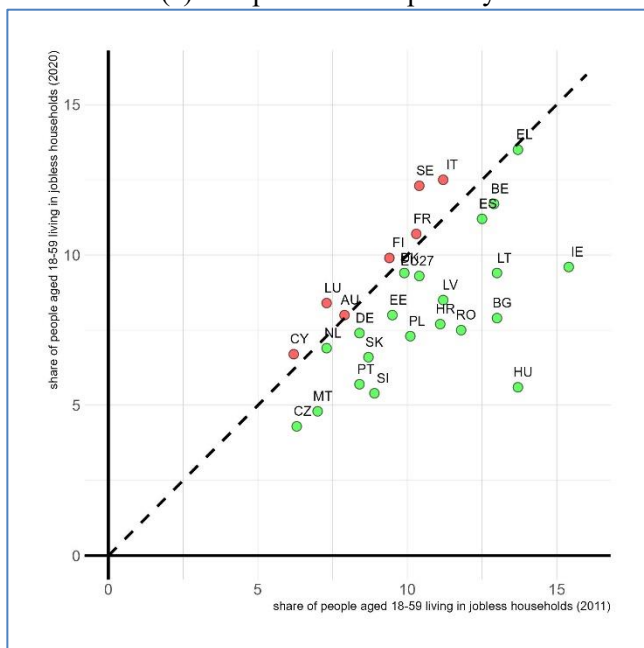
Figure 3: Country performances in the four study variables, 2011 and 2020.



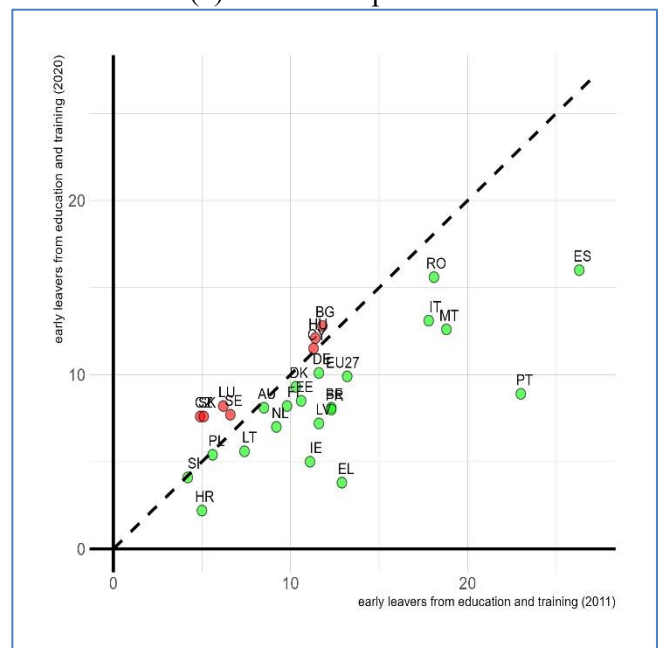
(a) People at risk of poverty rate



(b) Material deprivation rate

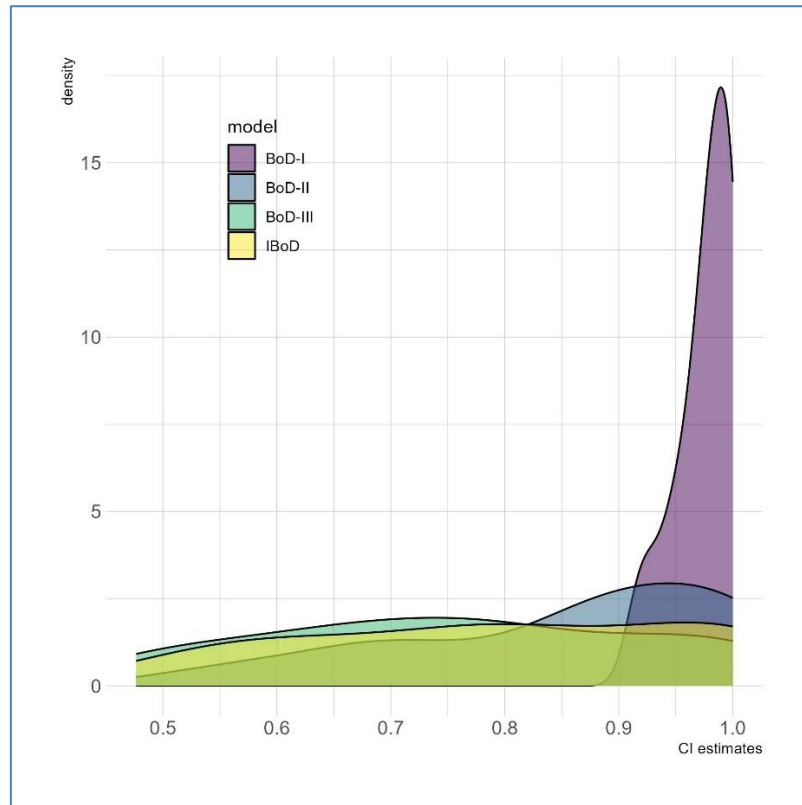


(c) People living in jobless households



(d) Early leavers from education and training

Figure 4: Efficiency distributions, alternative approaches for treating reverse social inclusion indicators, 2014.



Note: The IBoD model treats the indicators as reverse. The other three models transform the reverse indicators into forward using the min-max transformation (BoD-I), subtracting indicator values from 100 (BoD-II), and taking their inverse (BoD-III).

Appendix

Please see Table A1.

Table A1: Descriptive statistics of model variables

year	people at risk of poverty rate after social transfers	material deprivation rate	people aged 18-59 living in jobless households	early leavers from education and training
maximum				
2011	22.300	60.100	15.400	26.300
2012	23.100	61.600	17.600	24.700
2013	23.100	58.000	19.600	23.600
2014	25.100	46.800	18.800	21.900
2015	25.400	49.100	18.100	20.000
2016	25.300	46.900	17.100	19.000
2017	23.600	43.800	16.100	18.300
2018	23.500	33.600	14.800	17.900
2019	23.800	32.600	13.600	17.300
2020	23.800	32.700	13.500	16.000
minimum				
2011	9.800	4.700	6.200	4.200
2012	9.600	4.500	6.500	4.400
2013	8.600	5.400	6.200	3.900
2014	9.700	4.400	6.100	2.800
2015	9.700	4.000	5.800	2.800
2016	9.700	3.700	5.000	2.800
2017	9.100	4.200	4.000	3.100
2018	9.600	4.500	3.900	3.300
2019	10.100	3.700	3.800	3.000
2020	9.500	5.000	4.300	2.200
median				
2011	15.500	20.050	10.200	11.200
2012	15.700	20.750	10.350	10.400
2013	15.950	19.850	10.550	9.500
2014	16.550	19.450	9.850	8.750
2015	16.450	16.650	9.650	9.200
2016	16.500	15.100	9.600	7.950
2017	16.000	13.000	9.100	8.700
2018	16.400	11.300	8.300	8.450
2019	15.950	11.550	7.450	8.350
2020	16.100	12.450	8.000	8.150
standard deviation				
2011	3.392	14.349	2.438	5.432
2012	3.490	14.478	2.792	4.924
2013	3.619	13.815	2.857	4.705
2014	3.815	12.258	2.595	4.590
2015	4.010	11.993	2.538	4.266
2016	3.985	11.339	2.433	4.138
2017	3.954	10.516	2.330	3.825
2018	3.862	8.694	2.276	3.804
2019	3.868	7.844	2.190	3.615
2020	3.804	7.725	2.395	3.374

Note: all variables are measured in percentages of the total population except “people living in jobless households”, which is measured as a percentage of people aged 18-59.

Statements and Declarations

The authors have no relevant financial or non-financial interests to disclose.

CRedit authorship statement

Giannis Karagiannis: Conceptualization; Methodology; Writing - original draft;
Writing - review & editing

Panagiotis Ravanos: Conceptualization; Methodology; Writing - original draft;
Writing - review & editing

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Footnotes

¹ OMC is a voluntary coordination mechanism adopted for monitoring and evaluating EU country performances in policy areas over which these have no real competence and for which control over the policy schemes is maintained as the country level (see Atkinson *et al.*, 2004; Cherchye *et al.*, 2004). Its main features involve identifying best practices and disseminating national experiences between EU countries. (Rogge, 2017).

² The pledge was signed in the 2021 Porto Summit. The Porto declaration is available here: <https://www.2021portugal.eu/media/icfksbgy/porto-social-commitment.pdf>

³ This is the Eurostat Employment and social inclusion indicators database, available here: <https://ec.europa.eu/eurostat/web/employment-and-social-inclusion-indicators/social-protection-and-inclusion/social-inclusion>

⁴ In the literature associated with composite indicator construction, forward indicators are also referred to as indicators of “positive polarity” or “the-more-the-better” or isotonic indicators. Similarly, reverse indicators are mentioned as having “negative polarity” or as “the-less-the-better” or non-isotonic indicators (see, e.g., Diaz-Balteiro and Romero, 2004; Dialga and Ciang, 2016)

⁵ The nine material deprivation items in the 'economic strain and durables' dimension are: to pay rent or utility bills; to keep their home adequately warm; to pay unexpected expenses; to eat meat, fish or a protein every second day; a week of holidays away from home; a car; a washing machine; a color TV; or a telephone.

⁶ Rogge and Konttinen (2018) aggregated the same data for year 2014 using a variant of the BoD-I model. In that model, “pie-share” restrictions (see Cherchye *et al.*, 2007) were used to force the share of each of the four indicators in the composite index to be no less than 5%. After estimating this model, the composite index was constructed, for each country, as a geometric mean of its performance in each indicator divided by the overall EU27 performance, using the country-specific weights derived from the weight-restricted BoD-I model. The country ranking obtained differs from that in Table 3. In particular, although countries such as Sweden, Czech Republic, Denmark, and Luxemburg are ranked high, and Spain and Bulgaria are ranked low by both specifications, the BoD-I model used in the present paper indicates Croatia as a best-performing country and Italy as one of the lowest ranked countries, while the reverse is the case in the variant of the BoD-I model used in Rogge and Konttinen (2018).

⁷ The grouping of countries in Table 6 is the same as in Rogge (2017) and resemble the social democratic (Nordic countries), conservative/corporatist (Northern and Southern European countries), Anglo-Saxon/liberal (Ireland), post-communist and former USSR welfare state models. The only exception is that the UK (which was part of the Anglo-Saxon/Liberal model) is not included in the analysis of this paper.

⁸ On the other hand, the study of Rogge and Konttinen (2018), identifies Croatia as one of the worst performing countries, while Croatia is on the efficient frontier in our study.